

Data-spatialized Pavilion: Introducing a Data-driven Design Method based on Principles of Catoptric Anamorphosis

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Abstract

Data spatialization is a design technique through which data is used to create architectural spaces. It does not necessarily preserve the legibility of the represented data, but rather focuses on the spatial qualities that can be gained from the data. As a consequence, data in such data-driven spaces tend to be represented in abstract forms. By means of a method of spatial representation that has historically been used in art and architecture, we produce a data-spatialized architecture that preserves data legibility. This research aims to introduce a method for the design of a data-driven pavilion that represents data spatially through catoptric (mirror-assisted) anamorphosis. The major contribution lies at the underexplored intersection of data spatialization and perspectival representation, where the input data defines the physicality of the pavilion and simultaneously remains readable. In this work, a set of environmental datasets from North America – including elevation, precipitation, temperature, and population – is used to generate an anamorphic structure. The spatialized datasets can be updated by means of illuminating the components of the pavilion. Based on the result, this design methodology provides an accurate data representation in an anamorphic data-driven public space.

Introduction

Anamorphic projection attracts our attention by manipulating our spatial perception. As a subset of optically illusive perspectival representation, it highlights the gap between the representation of an object and its reality [21]. In other words, anamorphic projection has the potential to attract its audience and engage them in understanding the discrepancy between the representing and the represented [22]. What motivates this research is to harness the power of anamorphic projection in an architectural design using the aforementioned anamorphic attributes.

In architectural design, the advent of powerful computational techniques has opened up new horizons with the potential to develop new spatial and communicative possibilities by using “data” – i.e., *the input data* – as a design concept. This approach has led to the introduction of data spatialization, which provides architects with the opportunity to physicalize data at an architectural scale. Data-spatialized architectures, however, mandate that data be distributed within a three-dimensional space at an architectural scale [19]. Therefore, data in these spaces become abstract and are not as legible as may be desired. Such abstracted data may become hidden in the built environment, which renders the built environment mute when it could otherwise express itself in visually legible and socially meaningful ways.

To tackle this issue, this paper introduces a data-spatialization design technique that, while maintaining data legibility, employs data in the design of a pavilion through the use of catoptric (mirror-assisted) anamorphosis. The applied methodology maintains the two-dimensional representation of the input data within an anamorphic data-spatialized pavilion. It is based on dissecting the principles of a reversed curvilinear perspectival representation known as catoptric anamorphosis. This representation technique has historically been used to translate broad and often unperceivable images/data into a comprehensible form at the level of

small-scale objects. The objective of catoptric anamorphosis is to communicate wide-angle spatial qualities within a specific field of view. The role of the mirror is to reveal a hidden message (both two-dimensional and three-dimensional) that may otherwise be distributed and therefore illegible in the three-dimensional space.

In this paper, we establish a proof of concept by first designing an anamorphic sculpture of the Mona Lisa painting in a scaleless environment. Next, an architecture is introduced in the form of an anamorphic data-driven pavilion. Hypothetically, any easy-to-grasp type of data is applicable to this specific architecture. In this regard, we use environmental data since it can be understood by a broad range of audiences and engages with public awareness. Our data-spatialization process takes a series of environmental datasets of North America, including elevation, precipitation, temperature, and population data. Since elevation tends to remain consistent over time, it is used to build brick-like components mounted over the shell of the pavilion. The geometry of the shell of this pavilion has to be modified and improved in a way that it is fully reflected through the mirror. Such modification prevents any failure in maintaining the representation of data viewed through the anamorphic projection, and is handled by the design code. To provide an opportunity to update or swap datasets displayed by the anamorphic pavilion, all datasets, including the elevation data, undergo a process to uniquely illuminate every single brick of the pavilion. Such illumination helps in enhancing the dynamic ambience of this publicly-accessible privileged space.

Our main contribution is to introduce and develop a design methodology for a data-spatialized pavilion where the represented data remains legible, thanks to an anamorphic projection. This technique is expected to trigger public awareness towards the principles of catoptric anamorphosis, as a historical method of spatial representation in art and architecture. In a public space, such as the proposed pavilion, the audience would communicate with a series of represented environmental issues as a consequence of understanding how this projection works and how it is applied in building the pavilion.

Background

Designers can augment data-driven spaces in terms of legibility of the input data, manifested in the actual built environment. Within the scope of this research, this objective is accomplished by:

1. briefly reviewing the principles of catoptric anamorphosis projection and data visualization, and
2. employing data spatialization in a way that preserves the legibility of the input data in the built environment.

Toward this end, this paper focuses on three primary domains to investigate precedents and relevant projects: catoptric anamorphosis projection, data visualization, and data spatialization.

Anamorphosis

As an illusory technique of linear and curvilinear perspectival representation in the field of art and architecture, anamorphic projection highlights the gap between the physical form of an object and its visual appearance [21]. Wright describes anamorphosis as an illegible two-dimensional artform that becomes meaningful when viewed from one particular point in the space or by using a special device [29]. The special device is most commonly a cylindrical, spherical, conical or polyhedral mirror [7, 5] that captures data beyond a human's normal cone of vision [30, 28].

The objective of anamorphic design is not limited to creating a deformed object that is supposed to be read clearly from one viewpoint. In some cases, its aim is to reorient and/or reorder spatial elements to suggest a perfectly composed geometric space. One astonishing example is in St. Peter's Square in Rome where the application of anamorphic design overrides the perception of the oval composition of the space into that of a circle from a privileged point in the space [21].

Anamorphic projection has also been the subject of a number of research projects within the domains

of art, architecture and mathematics. For example, a refined technique for the design and fabrication of anamorphic projection on complex surfaces is introduced in the work of Di Paola *et al.* [5]. In addition, Jovanovic *et al.* [16] explore an application of robotic arms in automatically generating anamorphic structures from a two-dimensional image. Another work approaches the subject using a ray-casting technique, and provides a comprehensive framework to generate anamorphic effects [4]. Optically Illusive Architecture (OIA) takes an anamorphic approach toward architecture that suppresses the three-dimensionality of the space from a privileged point [9]. Through the lens of mathematics, Hickin [8] explains his method for creating two-dimensional anamorphic effects, bundled with an in-depth process to implement the catoptric anamorphosis. Likewise, a similar research project provides a method to generate anamorphosis for three-dimensional objects by utilizing a variation of a simple projective map known within the computer graphics literature as *collineation* [7].

Data Visualization

Modern uses of data must contend with a massive, heterogeneous, and dynamic volume of information produced as a result of an ongoing data deluge [12], extensive portions of which are difficult to understand and analyze in their raw format. However, the integration of human judgment together with visual representations of the data turns this data overload into an opportunity [17]. Such data should be transformed into graphical and visual representations, which are referred to as data visualizations [28, 17, 18]. Data visualization allows a broad range of users to understand information concealed within the data by providing mental models of information [17, 27].

Research shows that an appropriate method of visualization improves users' cognition in the perception and exploration of data [25]. It also helps improve the efficiency of information retrieval and memorability of data [14, 23]. A visualization, once paired with a three-dimensional space, can influence spatial perception skills. Moreover, it has the potential to expand the public's exploration and understanding of critical, complex data via inclusion of data in sculptures and architectural installations [15].

Data Spatialization

As computational methods continue to develop, architects are taking advantage of the many data streams available to them during the design process [3, 6]. Using computational design tools to generate novel architectural forms and spatial opportunities bridges the field of data visualization and architecture, and forms the basis of data spatialization [19, 10]. Such representations of data additionally have an aesthetic appeal as an external aid for visual thinking [13], and are referred to as data sculptures [13, 20]. These data-based physical artifacts aim to augment the audience's understanding of data and any socially relevant issues that underlie it [31].

Several important preceding works have enriched the initial research and design development of this paper, with emphasis on catoptric representation, data visualization, and data-driven spatial effects. For instance, *Cloud Gate* in Chicago, Illinois, by Anish Kapoor is a sculpture that represents the surrounding buildings and the city's skyline, however, in a distorted way (Figure 1a). As far as this research is concerned, *Cloud Gate* triggers a critical question with regards to how its geometry contributes to the deformation of the cityscape. In other words, how should the geometry be designed if it is supposed to flawlessly represent its surroundings?

Living Light by David Benjamin, The LivingTM, is a sculptural canopy in a public park in Seoul, South Korea, that displays air quality in the region. This project aims to raise public awareness by combining real-time data and dynamic lighting. At any given time, if the day's air quality is better when compared to the previous year, a panel on the sculpture corresponding to the region is illuminated [2] (Figure 1b). *Living Light* raises a key question in terms of the role of data in forming the physicality of the canopy, calling into question whether the input data necessarily limit the variation of the geometry of the canopy.

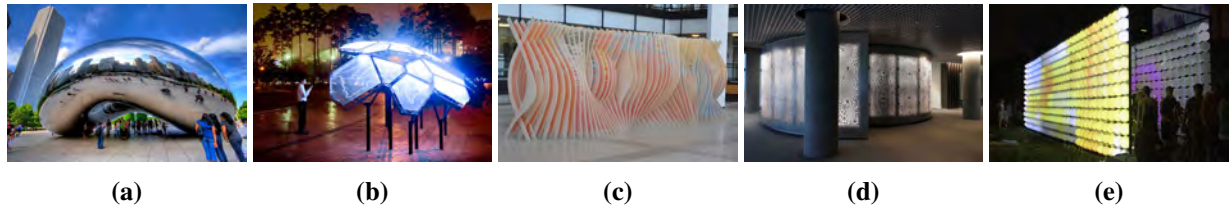


Figure 1: From left to right: (a) *Cloud Gate*, Anish Kapoor, 2006. (b) *Living Light*, The Living, 2007. (c) *Centennial Chronograph*, Marcus, 2014. (d) *Data Moiré*, Alvin Huang, 2016. (e) *Weather Report*, Swackhamer, 2017.

Centennial Chromograph is also a project in data spatialization, which “oscillates between representational and atmospheric readings” i.e., it occupies both the role of communicative and sculptural installation [19] (Figure 1c). However, this project represents an abstract form of data, and does not support updates to the data and interaction possibilities for users. *Data Moiré* uses the same approach and merges the territories of data spatialization in order to articulate a vast quantity of data as a spatial experience [10] (Figure 1d). Although the resulting project provides an architectural feature that enhances the spatial experience, it does not provide a readable representation of the input data to its audience.

Weather Report [24] is another example of data spatialization that relies on a design driven by data and driven by users (Figure 1e). This project uses a set of two illuminated balloon walls, and serves as an example of a successful *democratization of visualizations* [12, 20, 31, 11, 26] in a spatial context. While one of the walls represents quantitative real-time weather data, the other gives its audience the chance to intuitively design a visualization based on their recollections and memories of the weather data, i.e. a qualitative representation of data. As a spatial structure, it provides a novel method for user interaction, however, the design could have been elaborated further in order to allow spatial relationships with its surrounding environment.

While the projects above aim to spatialize specific types of input data in accordance with certain design parameters, they tend to undermine one key factor: readability of the data. In other words, the resulting embodiment requires a description, label, or legend to be decoded and recognized. This research aims to address this shortcoming by designing an anamorphic pavilion whose spatially-visualized data is accurate, dynamic, and easily recognizable by its audience.

Methodology

This project reverse-engineers the reflection law by means of a Grasshopper[®] code to simulate catoptric anamorphosis. Briefly, the code takes a two-dimensional curve as its input and generates a corresponding deformed curve, whose original form is revealed when placed in front of a particular mirror. The next step is to generate two-dimensional catoptric anamorphic images. Finally, the code is developed further to convert images into three-dimensional anamorphic objects.

The first experiment to three-dimensionalize an image is conducted using the Mona Lisa painting. Afterwards the research proceeds on to environmental datasets from North America to represent different types of engaging information. The results are expected to be legible when viewed from a privileged space through a cylindrical mirror.

The Pseudocode

The law of reflection indicates that rays of light travel through linear paths. When reaching a perfectly reflective surface, they bounce back at an angle identical to the incoming vector with respect to the surface

normal at the point of intersection [5, 4, 8]. The code reverses this phenomenon by first receiving a privileged point, a surface to serve in the role of mirror, a 2D or 3D space where the anamorphic object is formed, and the input curve (Figure 2a). Next, it converts the input shape into a series of points. Having placed the input shape between the privileged point and the mirror, the rays, extending from the viewpoint to the shape points, intersect with the mirror. In other words, the mirror acts as a *picture volume* [9] that hosts the intersection points (Figure 2b).

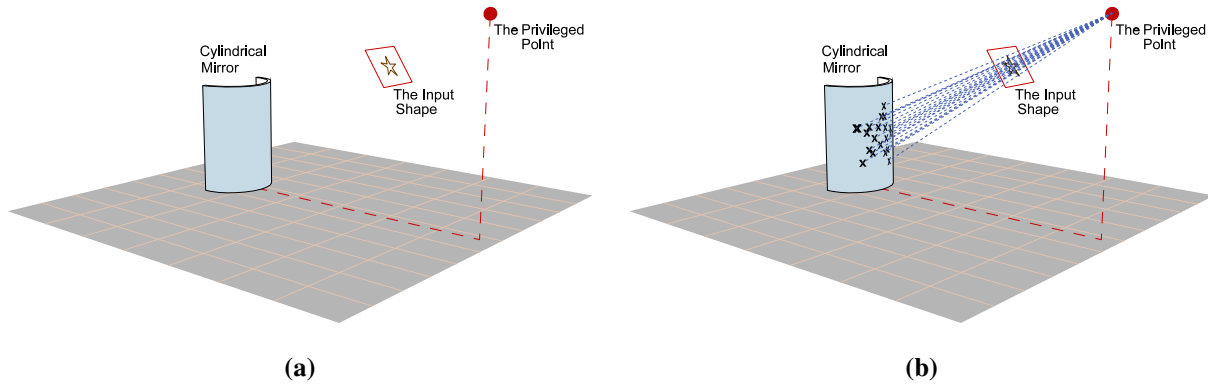


Figure 2: (a) The scene setup with the privileged point, the input shape, and the cylindrical mirror. (b) The input shape converted into a series of points and projected onto the mirror.

The law of reflection then allows the rays – extending from the privileged point to the corresponding intersection points on the mirror – to be treated as incoming rays of light and bounced/reflected off the mirror (Figure 3a). The intersection of these reflected paths with any arbitrary surface, i.e., the World XY plane here, constitutes a series of points. By interpolating a parametric curve through these points, an output shape is produced that reveals the original two-dimensional shape through the mirror when viewed from the privileged point (Figure 3b). In this project, we implemented interpolation through a Non-uniform rational B-spline (NURBS) periodic curve with a degree of three [1]. Interpolation through a higher number of points results in a higher quality of the anamorphic effect, once reflected on the mirror (Figure 4).

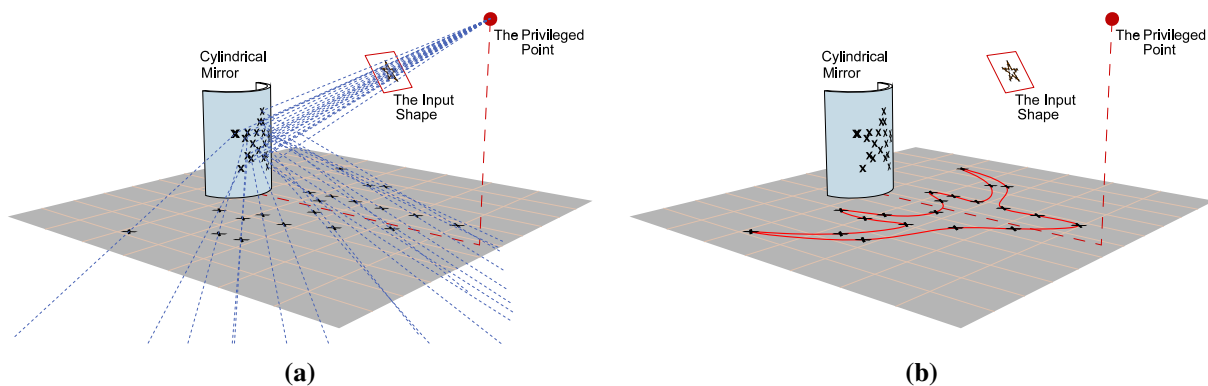


Figure 3: (a) The incoming rays to the privileged point reflected by the mirror. (b) The output curve generated by interpolating through the intersection points.

The code is then developed to take images as input, instead of curves. Accordingly, we import an image (Mona Lisa) into Grasshopper®, and divide it into an arbitrary number of cells. Needless to say, a higher number of cells generates a higher resolution result. At this point, the program treats each cell as a curve for

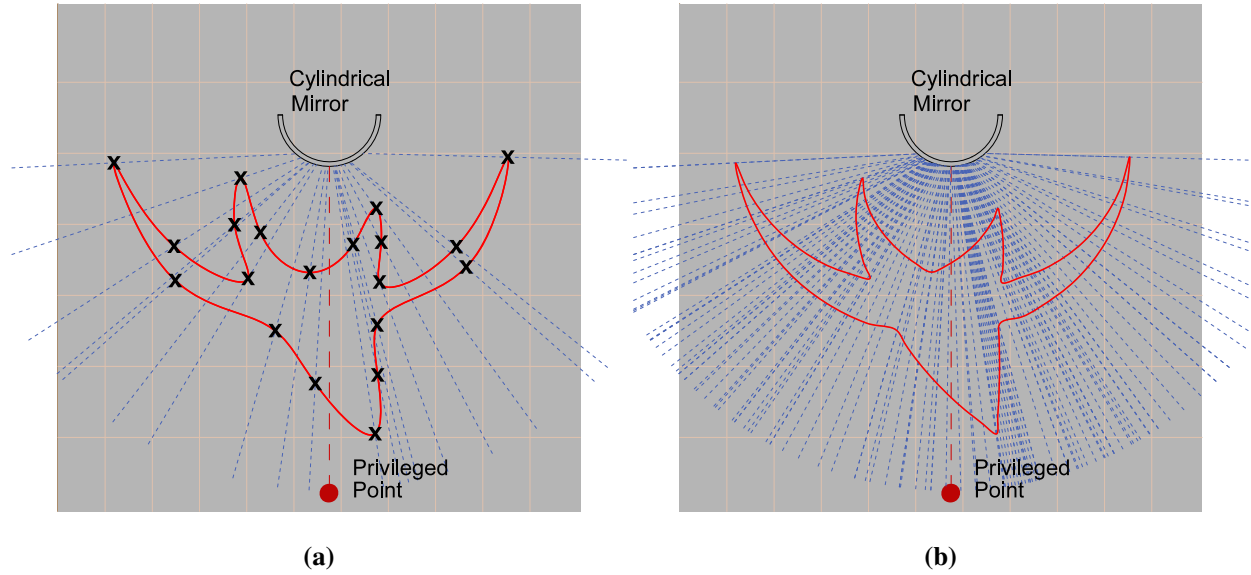


Figure 4: The output curve generated by interpolating through the intersection points. More points result in a higher resolution output shape. The anamorphic star has been generated through (a) 20 points, and (b) 100 points.

use with the first part of the code and ultimately generates the corresponding curve on an arbitrary surface, e.g., the world XY plane here. Next, the code extracts the average RGB color values of each cell and assigns the proper colors to the output cells (Figure 5).



Figure 5: A catoptric anamorphic image of Mona Lisa.

To generate a three-dimensional model from this result, we use the RGB value of an input cell as an extrusion factor for the corresponding output cell. The RGB values are converted to grayscale ranging from 0 to 255, i.e., black to white, meaning that cells with smaller RGB values receive less extrusion and are, thus, shorter.

Moreover, to avoid failure of the extrusion process, cells with the RGB value of 0, are assigned a small number as their extrusion factor. Figure 6 illustrates this method when the extrusion factor is applied along the Z vector.



Figure 6: The three-dimensional object corresponding to the Mona Lisa painting. Isometric and top views of (a) the virtual model, and (b) the physical 3D-printed model.

Likewise, in catoptric anamorphosis, the extrusion is assigned a direction such that the two-dimensional image is displaced in a third dimension that gives it spatial depth. Per the first part of the code, every individual cell of the image correlates to an interpolated cell on the selected surface, i.e., the world XY plane. The extrusion direction of every single cell is a vector extending from the cell center, on the world XY plane, to the corresponding cell center on the mirror. Figure 7 illustrates the extrusion vectors in a conceptual four-cell image, and Figure 8 shows the results of this approach to three-dimensionalizing the Mona Lisa painting.

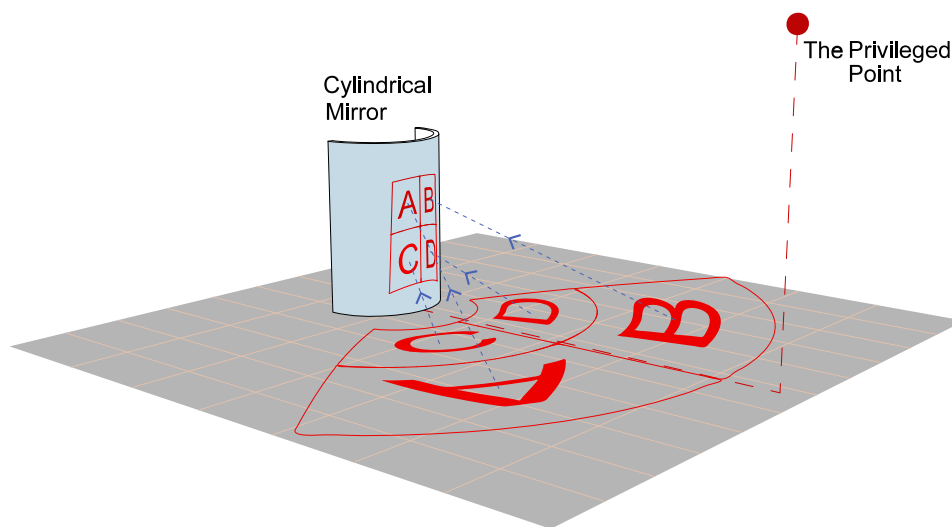


Figure 7: A sample four-cell image and its extrusion directions.



Figure 8: *A three-dimensional catoptric anamorphic representation of the Mona Lisa painting. (a) The virtual model. (b) The virtual model with the RGB values assigned to the ends of the extruded cells to enhance the effect. (c) The 3D-printed prototype.*

Results

This data spatialization project uses the method of catoptric anamorphosis and aims to translate widely distributed data into a medium that is easily readable by its audience and represents a spatial relationship between a pavilion and its urban environment. We conclude our project by introducing a data-driven designed pavilion with a mirror-based anamorphic structure (Figure 9).

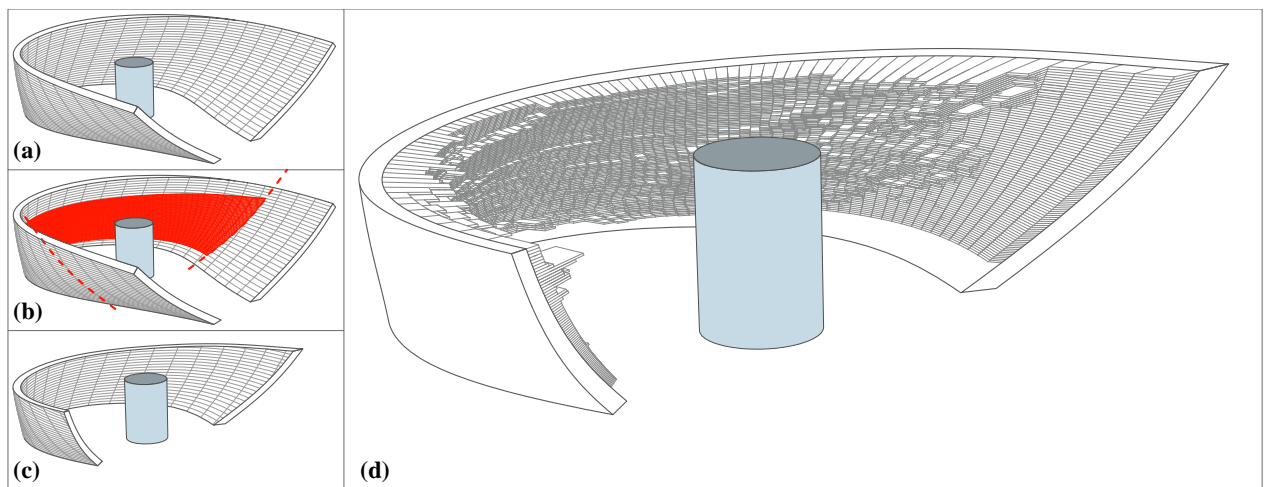


Figure 9: *(a) The host surface, (b) the locus of the anamorphic refraction, (c) the modified subsurface cut-off from the host surface, and (d) the pavilion with brick-like elements mounted over the host surface.*

As mentioned earlier, we tend to use environmental datasets since we believe this type of data speaks to a broader range of audiences. In pursuit of this goal, a system is used to retrieve the datasets corresponding to the desired region of interest, together with live updates or changes in the data through a specified period

(<https://www.globalgridsystems.com>). In our project, the region of interest is North America, and the desired datasets – in the form of 2D images – include elevation, precipitation, population, and temperature data.

Among the retrieved datasets, the elevation data is chosen to build 6400 brick-like elements mounted over the host surface. The reason we choose elevation as the primary source of data to form the structure is that it is relatively consistent through time. These bricks will also be overlaid with updatable datasets of elevation, temperature, precipitation, and population in order to create a dynamic data spatialization. We propose this dynamism in the visualization of data to be supported by illuminating the top ends of each brick using configurable LED lights. This creates a unique projection method that not only supports the dynamic representation of data, but also serves as a lighting design for the anamorphic structure that is capable of rendering a different dataset each time. The results establish a high level of data-legibility in this data-driven pavilion, thanks to the anamorphic projection (Figure 10). Since each brick represents a specific portion of the data, a higher resolution result is achievable by assigning a smaller portion of the data to every single brick, and, therefore, increasing the number of bricks mounted over the shell of the pavilion.

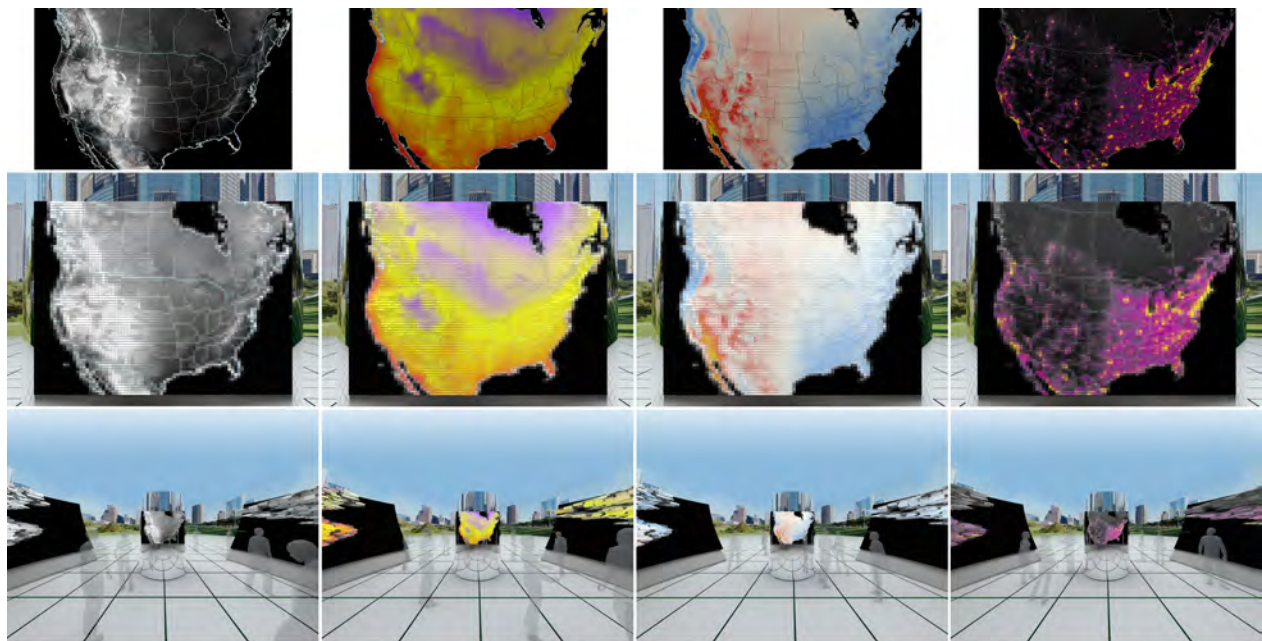


Figure 10: *The anamorphic projection maintains the readability of the input data in the data-spatialized pavilion. Top: the input data. Middle: the representation of the corresponding data in the mirror. Bottom, from left to right: pavilion representing elevation data, temperature data, precipitation data, and population data, respectively.*

Discussion

The prime objective of designing this anamorphic pavilion is to keep the input data as legible as possible within the built environment while allowing people to interact with the space. While harnessing an optical illusion, it defines a notion of playful urbanism where people feel there are discoveries to be made. In this regard, we shall sacrifice one goal to save a more significant one. For instance, the set-up of the pavilion, including its geometry, radius of the mirror, and the height of the privileged point, pilots the anamorphic effect to remain accessible and, more or less, form within human reach. Indeed, this is the key to framing such a structure as an interactive space. By means of interaction, audiences discover that they can disrupt and/or augment the illusion by the presence of their own body in the space. Nonetheless, partial occlusion becomes inevitable as a consequence of audiences interacting with the space between the structure and the

mirror. Figure 11 illustrates an alternative set-up with an overhead locus of the anamorphic effect to avoid occlusion. However, we intentionally designed the pavilion to let the audience interact and learn about this historical method of projection, at the cost of occlusion. Furthermore, if the only purpose of the design was to inform the public of the input data, a high resolution screen would arguably be the easiest solution.

Fabrication, as another concern associated with this structure, is entirely dependant on materiality, the actual scale of the pavilion, and the accessible tools of fabrication. It is within the future scope of this research to study the pavilion through the lens of tectonics and how the components could possibly mount over the structure once physically fabricated. After finalizing the materiality, the scale, and the tools of fabrication, two critical questions from the digital-fabrication side of the project (as a future work) can be the following.

1. What are the precise dimensions in every individual cross section, where the geometry is changing in support of producing the optical effect?
2. How to dissect the entire structure into small parts to make the pavilion easy to assemble?

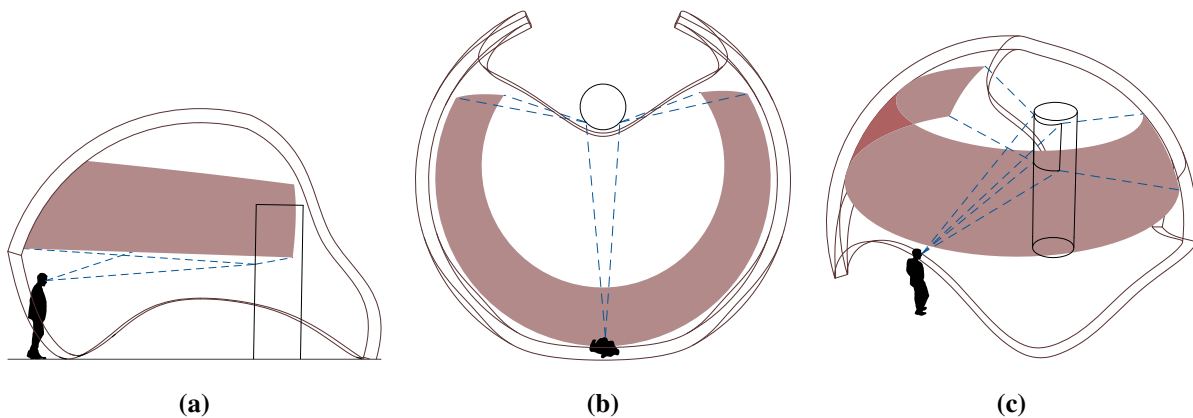


Figure 11: *Although the overhead alternative eliminates the occlusion, it mostly stays out of reach and does not provide much interaction.*

Conclusion and Future Work

This investigation reaches territories beyond a combination of a simple gimmick, data visualization, and data spatialization. It encourages the audience to discover spatial agencies and orientations through a traditional method of representation, i.e., catoptric anamorphosis.

This pavilion is a spatial structure that represents data in a readable non-abstract format and interacts with its audience and its environment (Figure 12). The proposed methodology allows us to harness data and catoptric anamorphosis to design a publicly-accessible privileged space at an architectural scale.

Despite the fact that the anamorphic effect is designed to be perceived from a unique privileged point in the space, it still seems to be fully graspable within an area adjacent to the privileged point. Defining the threshold of this area where the effect is still recognized could be accomplished through a user study. Moreover, offering control of the illumination to the audience could form the basis of another user study on the concept of democratization of visualizations in data-driven spaces. The aforementioned studies can be conducted via Virtual Reality (VR) as a future work. Users will be asked to virtually navigate within the pavilion and provide feedback on various subjects. For example, locating an optimized privileged point with minor occlusion, gaining knowledge on how the anamorphic effect works, etc.

Based on the results of these VR studies, a refined design for the pavilion can undergo a fabrication process at its actual scale, which provides the chance to consider this project as a behavioral-design objective. In

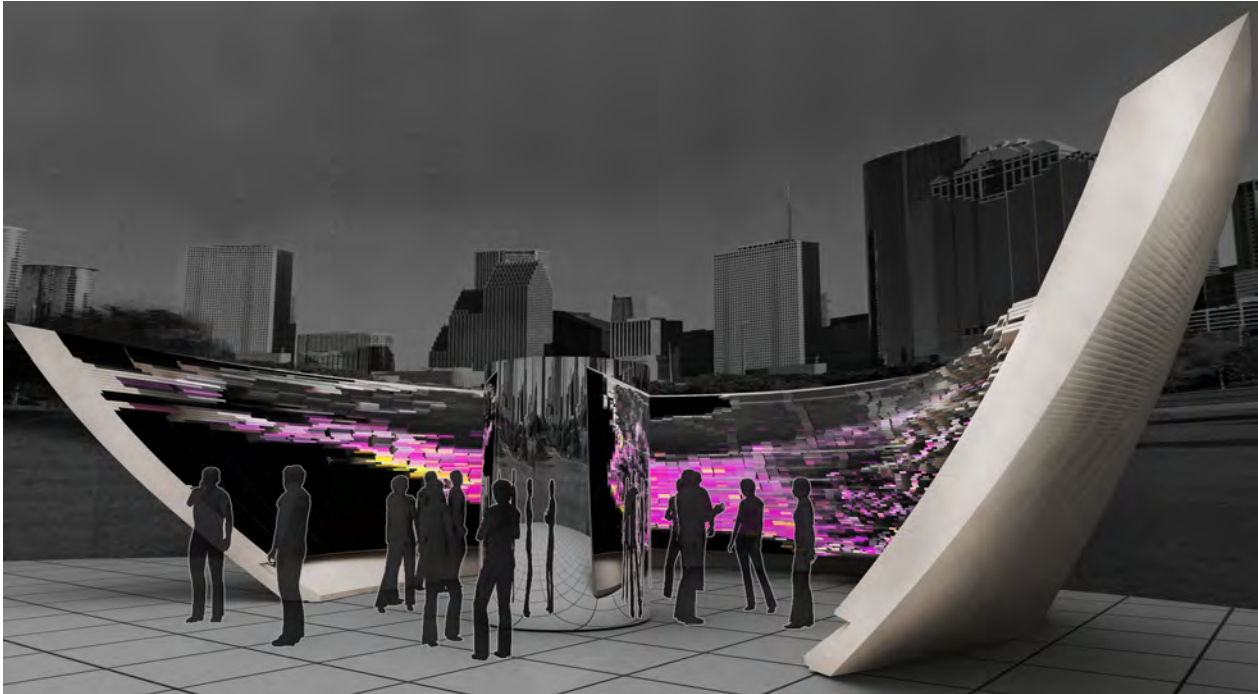


Figure 12: *The catoptric-anamorphic pavilion.*

this regard, research will be conducted to evaluate the potential to foster sustained curiosity, awareness, and discovery within the built environment that may otherwise lack interest. Such an approach creates interaction possibilities for users by encouraging them to engage with the design and become part of it while reading and understanding the data.

Image Credits

- Figure 1(a): Cloud Gate, by Craig Sinclair, <https://www.flickr.com/photos/craig-sinclair/2626662070>. <https://www.flickr.com/photos/craig-sinclair/>, CC BY 2.0, via Flickr.com.
- Figure 1(b): Living Light, photo courtesy of The Living <http://www.thelivingnewyork.com/>.
- Figure 1(c): Centennial Chromograph, photo courtesy of Variable Projects <http://www.variableprojects.com/>.
- Figure 1(d): Data Moiré, photo courtesy of Synthesis Design + Architecture, <https://synthesis-dna.app.box.com/v/IBMSF>.
- Figure 1(e): Weather Report, MINN LAB Design Collective, by Krista McCullough (used with permission).

All other drawings and images by the authors.

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