Art and Nonlinear Projection

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Abstract

Nonlinear projection is a current area of research in computer graphics and provides a meeting place between science and art. We begin by examining several cases of how and when artists choose to use nonlinear projection as opposed to perspective projection. We then provide a brief review of different techniques for nonlinear projection described in computer graphics literature. We expand upon our nonlinear projection framework: Flexible Projection [3]. Lastly we describe how Fleixble Projection was used in a collaborative project between an artist and a computer scientist.

1   Introduction

One of the most difficult aspects of creating images is determining how to represent three dimensional scenes on a two dimensional medium. This process of moving from three dimensions down to two dimensions is known as projection and it is not easy. The specific type of projection (if any) and our adherence to this type can drastically affect the impression an image makes. Some projections can make images seem very realistic; others can make the images surrealistic. Some images make you feel like you are within or a part of the image, other images clearly express a divide between you, the viewer, and the scene being portrayed.

Projection is an interesting meeting point between art and science. The scientific approach to projection is to view it as the mathematical reduction of dimension and organization of the reduced dimension's space. From here formulae, geometries, and spaces are theorized and derived. Artistically projection is a means of organizing space, relationships between objects, and the observer's presence in an image.

By far the most common type of projection is perspective projection, this is the type of projection produced by most cameras and is seen widely in Western art. However both before and after the discovery and formalization of perspective projection artists have made use of other types of projection.

In computer graphics projection has mostly been limited to projections that are linear in nature; that can be represented as linear transformations. The two common examples of such projections are perspective and orthographic. Recently a number of works have begun exploring possibilities for nonlinear projection.

In this paper we begin by describing some current computer graphics approaches to nonlinear projection. Then we look at some examples of the use of nonlinear projection in art and discuss how these projections can be reproduced with computer graphics techniques. We move on to describe our nonlinear projection framework, Flexible Projection [3] in detail. Lastly we describe an art project that makes use of Flexible Projections.

2   Nonlinear Projection in Computer Graphics

Salomon [11] describes a wide variety of nonlinear projections including a variety of panoramas as well as fisheye, false perspective, telescopic, and microscopic projections. Wyvill and McNaughton [17] describe projections in ray-tracing as a mapping between the image space $\Re^2$ and a space defining rays $\Re^5$ (three coordinates to position the ray and two angles to orient it). They demonstrate how this mapping can be manipulated to produce fisheye and panoramic projections. Similarly Glassner [6] defines ray tracing projections with two NURBS (non-uniform rational b-spline) surfaces; one surface positioning they ray's origin, the other surface providing direction.

Nonlinear ray-tracing allows light to follow curved paths based on vectors fields [7, 16]. These projections have been used to visualize the geometric behavior of dynamical systems [7] as well as visualize relativistic effects [16].

Yu and McMillan [19] describe a framework capable of reproducing a wide variety of linear projections. Subsequently they provide a framework [18] for creating nonlinear projections by using many of these linear projections together to create a single image. Image continuity is maintained by careful construction and selection of neighboring projections. This can be considered a multi-camera projection.

Multi-camera projections combine images produced by several cameras with perspective projections into a single image. The simplest example of this is putting together a collage of images such as that expertly done in David Hockney's *Pearlblossom Highway No. 2*, 1986. Examples in computer graphics that make use of this sort of technique include Cubist Style Rendering [5], and Photographing long scenes with multi-viewpoint panoramas [1]. Rademacher and Bishop [10] provide an interesting variation where an image is created by a camera along a path. At regular points along this path columns of pixels are captures and placed beside one another to form an image.

A more advanced multi-camera approach is to replace combining images with combing objects in normalized device coordinates after projection. Agrawala et al. [2] do exactly this allowing each object (or group of objects) to be individually projected by a different perspective camera. These projected objects are then combined in normalized coordinates where depth testing and clipping operations occur. Singh [13] expands upon this, allowing objects to be projected by more than one camera at the same time. Objects projected by more than one camera are actually only projected once, by a camera that interpolates between the settings of all cameras operating upon the object. This work is expanded upon to constraints to the projection equations that greatly enhance scene coherence [4] as well as to explore interfaces to more easily control cameras.

A last example of a multi-camera projection is that of Trapp and Döllner [14] where planar projections and non-planar projections are combined. This combination occurs by placing tiles on that cover the image. Each tile corresponds to a planar or non-planar projection.

The Flexible Projection Framework [3], discussed in more detail in the next Section, operates by using surfaces to parametrically define a viewing volume. This framework is able to reproduce all of the previously mentioned projections that incorporate a single viewing volume thus excluding most multi-camera techniques.

Figure 1: *Vincent Van Gogh's* Bedroom at Arles, *1888. Subtle features such as the flooring pattern, the unusual angles at which the pictures on the wall, and the non-perpendicular appearing intersection of the walls have led Heelan [8] to conclude this image was constructed with a nonlinear projection.*
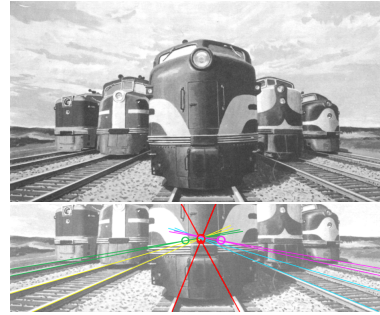

Figure 2: *Top: a painting for an advertisement for the Association of American Railroads. Image from [15]. Bottom: an image where we have extended lines from each pair of rails to examine the vanishing points. If all the rails are parallel then all the rails should converge at the same point. The colored circles highlighting the exact locations of the intersections show this is not the case.*

3   Nonlinear Projection in Art

Nonlinear projection has been used widely in art beginning with cave art and continuing until the present day. In this section we investigate a few key examples of nonlinear projection in art. We have chosen these examples as they provide insight on why and how artists choose to use nonlinear projection in their creations.

Nonlinear Projection from Optics. In *Secret Knowledge* [9] David Hockney examines a variety of evidence that leads him to conclude that many artists used concave mirrors and later lenses to construct images from the early fifteenth century and onward. While these primitive lenses would allow artists to paint from a moving image in bright light conditions a key limitation was the small size of the projected images. In general these images could be no larger than 30 cm across [9].

As a consequence if an artist wished to create a larger image the artists would either have to try to repaint the image larger without aid of projection, or project the scene a piece at a time and then blend the pieces together into a unified whole. Images constructed in this fashion no longer strictly adhere to perspective projection. Rather each object appears to be directly viewed (i.e., projected as if positioned in the center of the field-of-view of a perspective projection). Hockney [9] also points out that these composite projections, because all parts of the scene are viewed straight on, present a compelling closer view of the world.

It seems quite clear that this type of nonlinear projection performed by artists is the inspiration for the multi-camera projection systems discussed in Section 2. As this technique was originally created to bypass the technical restrictions of early optics one wonders if this truly is a good technique for composing nonlinear projections. On the other hand these techniques tie their approach to perspective, are based upon a clear, physically metaphor (that of constructing a collage), and lastly usual involve composing the projection based on image space.

Van Gogh. Heelan [8] proposed that many of Vincent Van Gogh's paintings from 1888 on portray the subjective perception of constant curvature hyperbolic geometry proposed by Lunenburg. This projection is divided into three zones: near, intermediate, and far. In the near zone vertical and horizontal surfaces appear to protrude in a convex bulge and edges close to the observer will be distorted into a convex arc. In

the intermediate zone will appear much as they do in a perspective image but in the far zone vertical and horizontal surfaces will appear to be concave and depth differences (e.g., foreshortening) will become less noticeable or absent [8]. A key overall difference is that the horizon will appear a finite distance from the observer, rather than at infinity as in a perspective projection [8]. Figure 1 shows Van Gogh's *Bedroom at Arles*, 1888 that demonstrates these characteristics. Heelan [8] maintains that Van Gogh's paintings provide a visual embodiment of the subjective spatial perception of binocular vision.

To examine how this sort of projection could be handled with currently existing computer graphics techniques, the key point is that the projection changes between the foreground, middle, and background. Such a projection could be modelled with the curved viewing volume of Flexible Projection or it a unique projection equation could be derived from geometry that Heelan has described.

Creative Perspective. Watson's book, *How to use Creative Perspective* [15] provides interesting insight into the many techniques used by illustrators to create perspective projections as well as to modify them for various purposes. He calls such modified projections, creative perspective and notes that an artist who uses "perspective creatively [can] bend it to his uses rather than be limited always to strict conformity" [15]. These manipulations are used to: allow the viewer to see structures that should be hidden, changing the scene (usually by manipulating vanishing points) to improve its overall composition, allow the viewpoint to shift to more closely recreate human visual experience, and lastly make the illustrations more dramatic and interesting. This last point, while somewhat vague is important. Manipulating the projection allows illustrators to present objects in precisely way that they feel will have the desired impact.

We will examine two examples from this text. The first is the illustration of trains shown in the top of Figure 2. The bottom of this figure shows an analysis of the vanishing points along the rails. According to Watson [15] the artist has purposefully introduced this distortion to the projection to compensate for eye movement across the picture and make the scene seem more natural.

The second example, shown in Figure 3, shows how objects within the scene have been moved and reoriented for effect. Looking at this figure we can see that the illustrator has slightly altered the orientation of the wagon in relation to the horses as well as the orientation of the horses to one another in order to present the best, most dramatic views of each [15]. With these subtle changes the illustrator has maintained realism while improving the produced image.

These two examples are difficult to describe as being best handled by any single computer graphics nonlinear projection technique. The first example can be achieved with a multi-camera approach but care needs to be taken to achieve a smooth transition between changing viewpoints. Flexible Projection could also be applied; for this continuity is easily achieved but adjusting the volume to achieve the exact change in vanishing points requires finesse. The second example is even more interesting in that a kind of nonlinear projection is achieved through adjusting the model and not truly requiring nonlinear project at all.

4   Flexible Projection

Flexible Projection [3] is unusual as it approaches projection, a rendering technique, with modelling tools. The primary concept of Flexible Projection is in defining the viewing volume (the 3D volume that will appear in the 2D image) as a parametric volume $Q(u,v,t)$. This volume is defined so that the parameters u, v, and t correspond to the width, height, and depth positions that will result in the projected image. This volume can be stretched and reshaped to affect the projection and consequently the image.
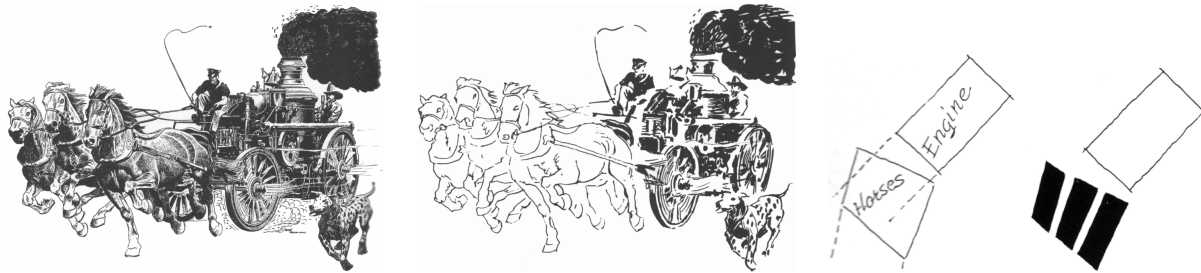
Figure 3: *Left: illustration by Fred Freeman. Middle: sketch by Watson presents the illustration as if wagon and horses are travelling in the same direction in perspective. Bottom: diagrams of the horses' position relative to the wagon from a top-down view. All images from [15].*
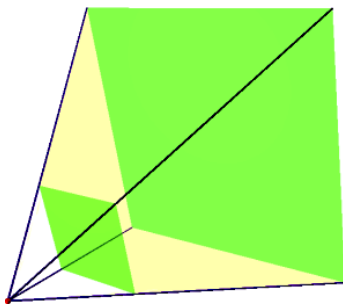

Figure 4: *A perspective projection's viewing volume. The viewing volume is bounded by the near and far planes (green) and the sides of the frustum (yellow).*
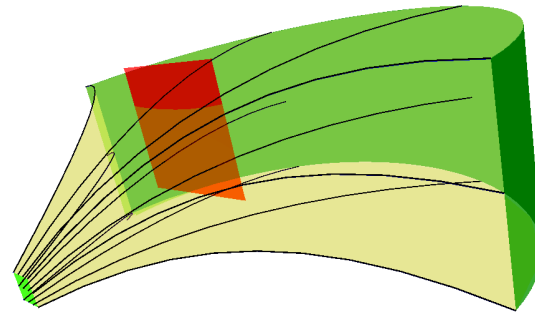

Figure 5: *The viewing volume of a Flexible Projection. The volume is shown in yellow, the green surfaces are the near and far surfaces, the red surface controls the curviness of the volume, and the lines shown projectors within the volume.*

While this volume can be defined in any manner whatsoever we recommend using familiar surfaces to aid in the definition. For example, consider perspective projection. This projection uses a frustum as its viewing volume and is usually defined by an eye position, near surface, and far surface as shown in Figure 4. Due to the familiarity of this construction it makes sense to define Flexible Projections by defining a near and a far surface. We can then create our volume by interpolating between these two surfaces:

$$Q(u,v,t) = (1-t)S_0(u,v) + tS_1(u,v) \quad u_{min} \leq u \leq u_{max}, v_{min} \leq v \leq v_{max}, 0 \leq t \leq 1$$

where S_0 and S_1 are the near and far surfaces respectively. We do not include an eye position as many projections do not feature a single eye position.

Interesting variations on projection can be obtained by altering the interpolation between surfaces. For instance, by adding surface(s) between the near and far surfaces we can use Bezier curves to interpolate between the surfaces obtaining greater control over the volume as is shown in Figure 5. This results in a definition of our volume as:

$$Q(u,v,t) = \sum_{i=0}^{n-1} B_{i,n-1}(t)S_i(u,v)$$

where $B_{i,d}(t)$ is the Bernstein polynomial $\begin{pmatrix} n \\ i \end{pmatrix} t^i(1-t)^{n-i}$ and S_i is the ith surface (making S_{n-1} the far surface). Certainly other curve schemes could prove useful for this interpolation, however Bezier curve's ability to interpolate the near and far surfaces makes this curve scheme a useful place to start. Figure 6 shows how a curved volume can be created to show the front, back, and side of a car at the same time.
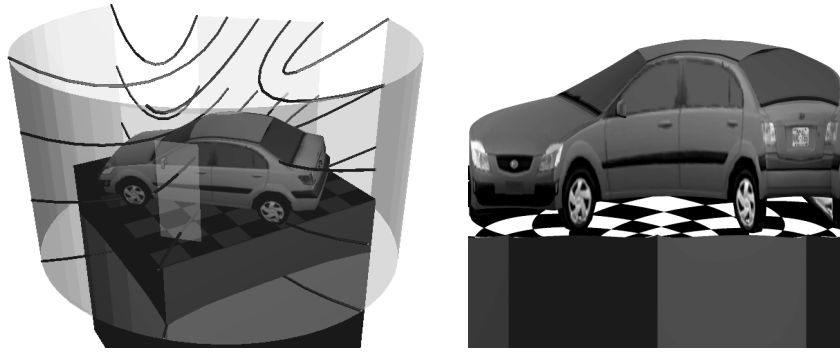
Figure 6: *Flexible projection using nonlinear interpolation. Left: shows the surfaces used to create the project. The large curved surface is the near surface, the rectangle in front of the car is used to curve the volume, and the rectangle behind the car is the far surface. Right: the projected result.*

Projection in the volume occurs based on surface parameterization. We can imagine projection occurring by following light paths through the volume. The paths the light follows are iso-curves within the volume, known as projectors, that are defined by fixing u, v values $Q(u, v, t) = Q_{u,v}(t)$. For surfaces that flatten easily image creation is a matter of scaling parameter values by the desired pixel dimensions of the image. For surface parameterizations that do not map directly to an image (i.e., the case of a hemisphere parameterized by polar coordinates) an additional step of reparameterization is necessary. This reparameterization step can additionally be used to apply distortions to the final image (e.g., zooming into particular areas, etc).

In implementation projection of these viewing volumes occurs in one of two ways. The first and simplest is through ray tracing. Upon determining the parameter values u_0, v_0 that corresponds to a particular pixel, the corresponding ray is the projector $Q_{u_0,v_0}(t)$. If the volume has not linearly interpolated the projector is not a linear curve and thus not suitable for the usual ray-tracing algorithm. There are two options to proceed, the first as Gröller [7] suggests is to break the curve into linear pieces and proceed with ray-tracing. The other suggested by Brosz et al. [3] is to limit our volumes to quadratic curves and compute object-quadratic intersections.

The other way of achieving projections is to derive projection equations. That is to determine the inverse of Q:

$$Q^{-1}(x, y, z) = (u, v, t).$$

In general this inverse is not always achievable. Fortunately there are several cases where the inverse is derivable. In particular if we can derive t directly from a distance measure (such as dealing with surfaces that are nested cylinders or parallel planes) and the parametric surfaces have linear parametric equations Q^{-1} can be obtained. These nonlinear projections equations can then be performed by vertex shaders to achieve realtime rendering.

5   Using Flexible Projection in an Art Installation

As a part of a collaborative course between the Alberta College of Art and Design (ACAD) and the University of Calgary Hao Wang and John Brosz set out to complete a mutually beneficial project. The project's goal was an art installation named *Perspectives* that aimed to explore the 2008 confrontation between Tibetans
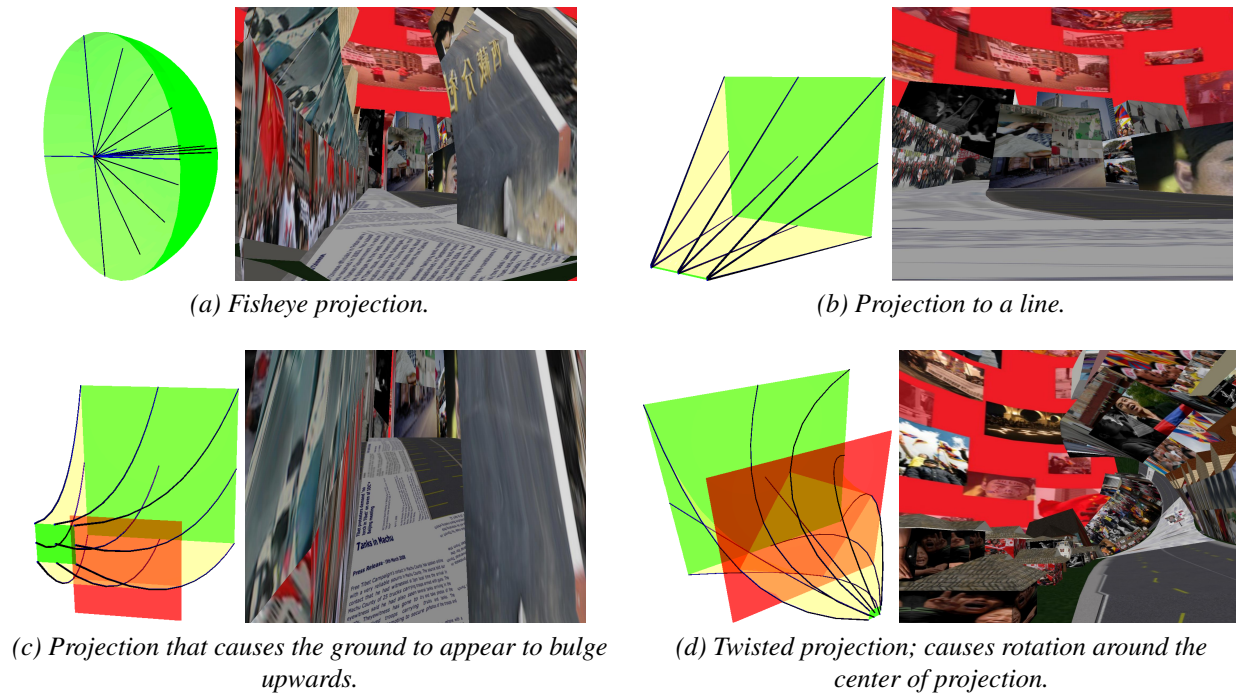
(a) Fisheye projection.

(b) Projection to a line.

(c) Projection that causes the ground to appear to bulge upwards.

(d) Twisted projection; causes rotation around the center of projection.

Figure 7: *Projection diagrams (left) and resulting images (right). In the diagrams the near and far surfaces are green, the viewing volume is yellow, and projectors are shown with lines. The middle surfaces in (c) and (d) are used to curve the volume.*

and the Chinese authority. In particular this project aimed to portray news images from distorted viewpoints representing the biases that exist in the media as well as between the involved parties.

Flexible Projection was useful in applying these distortions for two reasons. The first was they could be adjusted to affect different depths of the scene differently, as a result the background of the scene could feature a different sort of distortion than the foreground. Another interesting aspect was the Flexible Projections can be animated by moving the projection surfaces over time. This added an extra impression of change to the projection. Sixteen different projections were created for the project. Some examples of the fourteen non-animated projections are shown in Figure 7. One of the animated projections is shown in Figure 8.

The aim of the project was to create an interactive environment. The environment, a city setting, was textured with news stories and images related to recent events in Tibet and protests in Canada. The projection surfaces were translated and rotated with standard viewing transformations on a cyclical path through the environment.

Realtime rendering was achieved through the previously mentioned technique for deriving projection equations and implementation with GPU vertex shaders. Performance rates of approximately 25 frames per seconds on two display devices of 1280 by 1024 resolution were achieved with an Intel Core2 6600 CPU, an NVIDIA 7800 video card, and 2 GB of system RAM.

For the installation two modular ambient display screens [12] displaying different projections were placed side by side. A custom made button provided an interface to change the shown projections. The installation setup is shown in Figure 9 and was presented at the ACAD graduation show in May 2008.
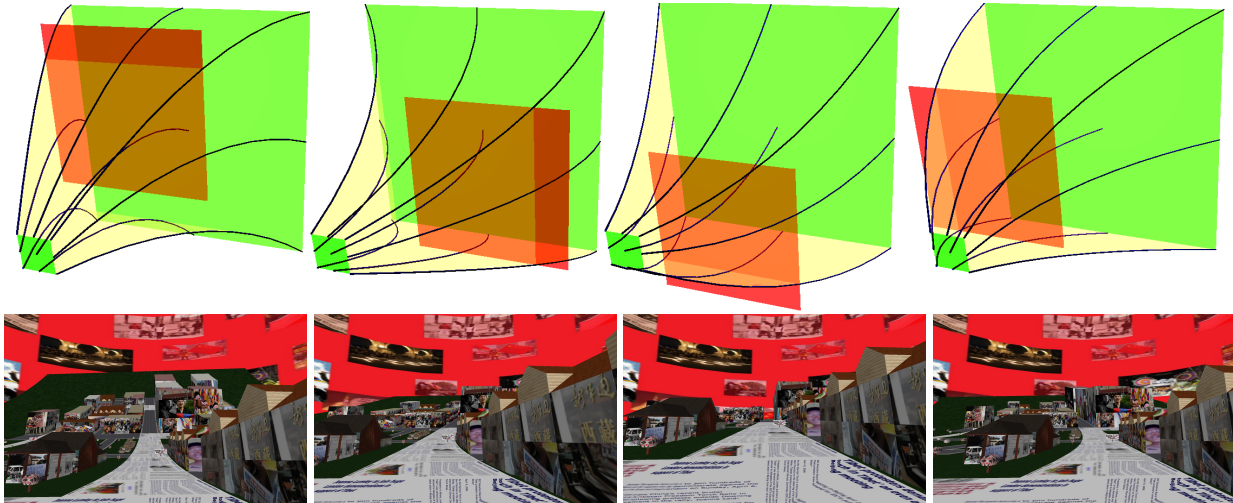
Figure 8: *Keyframes of an animated projection. The middle rectangle that controls the curving of the volume is moved in circles causing the middle ground of the produced images to bend in a circular motion.*
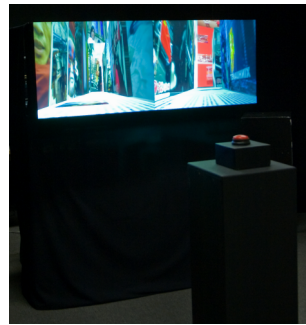

Figure 9: *The* Perspectives *art installation.*

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