Interactive Modeling of Muqarnas

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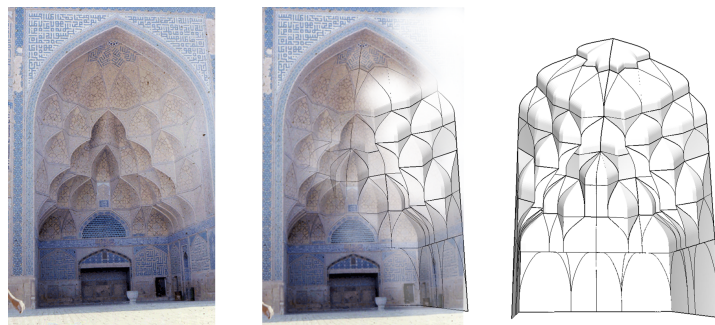[2]Department of Computer Science, American University of Beirut

Figure 1: *Model of an existing building.*

Abstract

Muqarnas is a mesmerizing 3D feature of Islamic architecture that exhibit intricate geometry. Such designs are composed of several basic structures combined in successive layers, producing complicated 3D surfaces. In this paper we propose a new approach for interactive modelling of muqarnas based on their layered structure. As a guidance for the modeling workflow, floor plans are used. We also discuss how to edit the resulting models and how to automatically generate new forms.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Modeling packages

1. Introduction

Mathematics and art, these two seemingly diverse aspects of humanity, have influenced each other from the early days. Over thousands of years, the use of geometric patterns as decorations is well established. Art has evolved and developed along with humankind, changing not only with time but also with cultural and geographical conditions, resulting in a diversity of styles. For instance, traditional western art exhibits figurative art which is derived from real object sources. In contrast, use of abstract geometrical patterns became a popular and principal characteristic of Islamic art simply because Islam does not encourage figure represen-

tations in artistic expression. Consequently, Islamic art has a rich mathematical foundation.

Islamic geometric decorations can be categorized into two major categories: two dimensional and three dimensional patterns. The first group is the type of ornament used to decorate flat or curved surfaces, such as domes or walls. A well-known example of this category is star pattern. The other category is Muqarnas, a type of 3D ornamental structure.

Muqarnas is the Arabic word for stalactite vault; a captivating structure based on replicating units arranged in tiers, each one supporting another corbel on top of it. The early purpose of these structures was to create an architectural transition between a circular dome and its square supporting

Figure 2: *Various examples of muqarnas. From right to left, top to bottom: portal, mihrab, dome, walls, flat ceiling, corners, base of a dome, niche and minaret [Tak10].*

structure. Later, muqarnas was used in different parts of a building, such as in large domes, cupola, niches, on arches, and as a virtually flat decorative frieze. Some examples of the different applications of muqarnas are shown in Fig.2.

Unfortunately, only few practitioners exist today who have learned the artistic skill of designing muqarnas from the older masters. Accordingly, muqarnas is in danger of extinction. Our main motivation in this paper is to revitalize the traditional art of muqarnas by analysing their geometry and by providing a set of methods to algorithmically model muqarnas. Such methods enable us to develop software tools for modelling muqarnas and consequently, preserve this cultural heritage for future generations. Moreover, modelling muqarnas is a valuable step toward better understanding of these complex geometrical forms.

In this paper, we introduce a new approach for modelling muqarnas interactively. Our paradigm is based on modelling the visible surface of muqarnas as a set of transitive layers, where each layer consists of a set of B-spline patches fit together seamlessly with possible sharp edges. Our approach benefits from the layered structure of muqarnas described in Section 3.1. We also take advantage of floor plans as a guide for the workflow of modeling. As discussed in Section 3, there are plenty of these plans available. The method we put forward is capable of generating a continuous space of muqarnas designs, therefore we are able to automatically create many new designs which can be used in modern architecture.

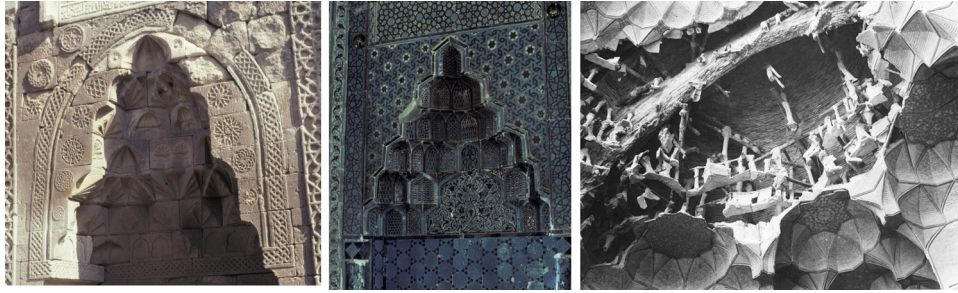We have implemented the proposed method as an in-

teractive modeling tool. This software tool can be used for the purpose of restoring existing muqarnas. There are many examples of muqarnas that have lost their geometric consistency after restoration [Tak10]. Thus, it is important that artists and architects attempting to reconstruct historical muqarnas have access to a virtual model prior to any restoration. As another application, this tool can assist researchers in recreating and visualizing 3D models of muqarnas from recordings of Islamic architecture. These recordings are usually in the form of maps or sketches (i.e. those recorded in the Topkapi Scroll [NaA95]). In the entertainment industry, our tool can be used to create a Middle Eastern atmosphere in games and movies. The animated film of Isfahan [Cri05] is an example of such applications of digital models of muqarnas. It can also be used to develop virtual tours for historical buildings that exhibit muqarnas. Virtual muqarnas models are also useful in pedagogical applications for architecture and history classes.

1.1. Construction methods

A good starting point to understand the concept of muqarnas is by understanding their construction methods. Muqarnas are typically constructed in three ways: corbeled, superimposed and suspended. A corbeled muqarnas is a thick structure built from stone or wooden blocks. Muqarnas cells are carved outwards from the middle of the block face, either before or after assembling. Fig.3 (left), shows an example of a stone corbeled muqarnas. To create a superimposed muqarnas, the supporting surface is built first (for example a dome or vault), then muqarnas elements are built up against concave surfaces, leaving no hollow space between the muqarnas and the supporting surface. Fig.3 (center), shows an example of a superimposed muqarnas. A suspended muqarnas is an assembly of surface-decorated panels. The panel components are produced on the ground, then combined and fixed to the architectural structure by means of attachment ribs. There is an empty space between the muqarnas shell and the structure, as shown in Fig.3 (right). Our approach in this paper is similar to the suspended construction method since we model the visible surface of muqarnas.

1.2. Challenges

Muqarnas in different historical periods exhibit various styles and designs. The characteristics of the materials and the cultural differences have a direct influence on this diversity. Both temporal and geographical diversity of designs require that our method supports a vast variation of designs. The lack of documentation from those who have designed and constructed muqarnas buildings is another issue. In few cases, there are documentations from traditional masters (e.g. [RM95]) but the descriptions and methods they provide are mostly informal, difficult to understand (due to their antiquated and specialized lexicon) and specific to few styles.

Figure 3: *Left: corbeled muqarnas. Center: superimposed muqarnas. Right: suspended muqarnas is process of repair. attachment ribs and empty space between the muqarnas surface and the walls are visible*

One of the most important characteristics of muqarnas is their rich symmetry. However, there is usually a level of inexactness and asymmetry in muqarnas maps and their actual constructions. This might be due to construction limitations or to the attributes of the surface on which a muqarnas was built. Yet some of this seeming inexactness might be a part of the design. The available maps of muqarnas, including those on-line [Tak10] and those in manuscripts of traditional artists (e.g. [RM95]), also contain a level of inexactness, which still might be intentional or result of errors in drawing. It is important to notice that even a small error in a radial pattern will cause a noticeable gap in the resulting dome due to replication. Therefore, creating exact models from inexact maps and buildings is one of the challenges we faced. At the same time, in most cases symmetry and repetition is not perfect. Symmetry might only be in one layer or in part of another. Moreover, it may contain exceptions here and there, which makes it even more difficult to deal with the problem of modeling muqarnas.

2. Related work

The problem of creating and designing two dimensional Islamic patterns (mostly star patterns) has been investigated in computer graphics [Kap08, Kap05]. In [GS93], the authors studied the formation of interlaced patterns from a small set of basic shapes. Rule of symmetry in Islamic patterns is investigated in [AS95] and also in [OH08]. In [WZS98], Wong et al. have proposed a method to automatically generate a western floral pattern in a closed area. Etemad et al. [ESP08] introduce procedural techniques for animation of Persian floral patterns. These patterns along with star patterns have been used to decorate muqarnas for several centuries.

Muqarnas are investigated in [NaA95] and [ACC07] from the architectural and historical aspects. From the geometrical point of view, the earliest work available is from Al-Kashi (1390-1450, Iran). In computing a module of the muqarnas, Al-Kashi decomposes a number of muqarnas forms into their basic units so that by rearranging those units different muqarnas designs can be formed [Yvo96]. As discussed in the introduction, this method has been used to construct some of the existing muqarnas buildings.
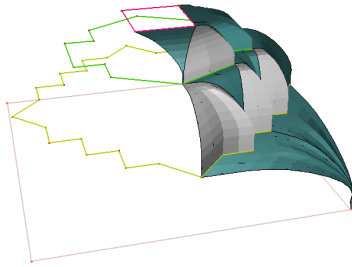
The floor plans of muqarnas have been used by traditional masters for many years (see [RM95] and [GO01]). Architects also use these plans in order to study muqarnas forms. Shiro Takahashi [Tak10] created a collection of plans of more than thousand buildings featuring muqarnas, classifying them into various types in an attempt to clarify the formative styles of muqarnas.

Using conventional CAD tools, a possible approach to modelling muqarnas in computer graphics is through solid modeling by creating a solid shape and subtracting sphere-like objects from it. However, shape, size and arrangement of the objects that should be subtracted is quite challenging. Furthermore, reshaping and editing of the resulting model is not simple.

Building muqarnas models from the arrangement of pre-designed units was investigated by Zellijs in [J.Z99] and also by Yaghan in [Yag01, Yag05, YH95, Yag10]. Yaghan proposed a set of primary units whose arrangement can create a specific type of muqarnas. Although this approach works for a class of muqarnas,it limits muqarnas form to combination of pre-designed units; which is considered a serious drawback, let alone the limited flexibility in creating new form. Moreover, the challenge of putting these units together seamlessly is still open.

Muqarnas forms provide an excellent framework for architects and designers to explore the design of new forms. From this aspect muqarnas has attracted the attention of many designers including western architects. Examples of this type of designs can be found in [CST11] and [Tib11].

Our method is not based on a limited set of building blocks. instead, we offer a continuous spectrum of designs which is richer than any reproduction of specific traditional examples. Such a space offers a great opportunity for architects and designers to explore new designs and artistic experiences. Using floor plans also eludes the need for a 3D imagination of the model. We support creating exact models from inexact plans. It also allows full interactive editing with

Figure 4: *Muqarnas as a transition between the layer lines.*

Figure 5: *2DPP of an existing building: Ivan at Masjid-i-Jame, Isfahan, Iran.*

Figure 6: *2DPP from [RM95]. The relation between two layers is shown in dotted lines. Note the inexactness of five point stars indicated by blue lines.*

minimum user effort. Moreover, we provide a computer generated texture atlas for each model which makes the texture design simple and precise.
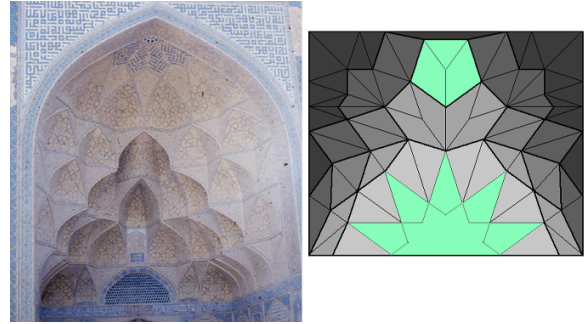
3. Interactive modeling of muqarnas

As described in section 1.1, modeling of muqarnas is a difficult task due to its complex shape, geometry and repetitive structure that creates specific symmetric patterns. The diversity of styles further increases the complexity of this task.
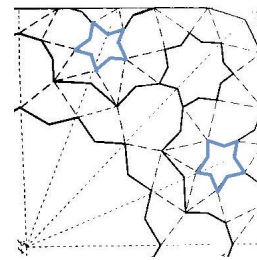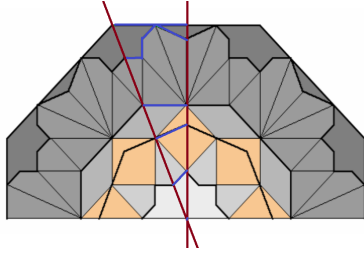
In this section we describe our modeling method. First, we discuss the geometry of muqarnas. Next, we explore floor plans, discuss our modeling workflow and editing tools. Finally, we present our method to automatically generate muqarnas forms.

3.1. Layered structure of muqarnas

From the geometric point of view, muqarnas is a transition between two shapes of different size, form, or position [M.A10]. For example, a muqarnas on the top of a column acts as a transition from a smaller to a larger circle. Similarly, a dome is a transition from a circle to a single point. This transition is usually done in more than one step. This means that the starting shape is converted to a number of other shapes before reaching its final shape (Fig.4). Each step in this transition is called a layer of muqarnas. The initial and final shapes of each layer are called layer lines. Traditionally, layer lines are poly-lines on horizontal planes arranged in ascending heights. According to the above definition, muqarnas is a set of surfaces that fill the area between consequent layer lines. The surface of each layer should interpolate the upper and lower layer lines. Every surface itself consists of smaller patches between the line segments of layer lines. Each patch connects a point or line segment in the lower layer to a point or line segment in the upper layer. The mapping between points on two layer lines is an important part of the muqarnas design. This mapping is not unique and has a major effect on the final form of the muqarnas. Vertical sharp features in muqarnas are a result of these connections whe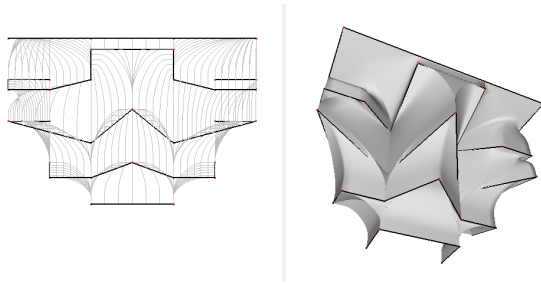re two adjacent patches meet. The profile of the surfaces that connects layer lines is also important. Depending on the design, the profile might vary from a straight line to a smooth or sharp curve and may exhibit horizontal or vertical offsets at each end.

3.2. Two dimensional pattern plans (2DPP)

As discussed in the previous section, muqarnas consist of layers laid upon each other. By discarding the height of each layer, they can be mapped onto a single plan. This mapping creates a ceiling plan as if looking at the muqarnas from the bottom upwards. These ceiling plans are referred to as two-dimensional pattern plans (2DPP). Fig.5 shows a muqarnas building and its corresponding 2DPP. In a two-dimensional projection, the layer lines may touch each other but they do not intersect. Since 2DPP cannot express vertical relationships between layers, a different method is usually used to indicate the vertical relationships between layer lines. For example, on some maps striped lines are used to show the correspondence between the points of two layer lines (Fig.6). Describing a muqarnas by its 2DPP was used by early practitioners. More than thousand of these maps are available on-line in [Tak10]. There are also books containing maps for existing and new designs (e.g. [RM95]).

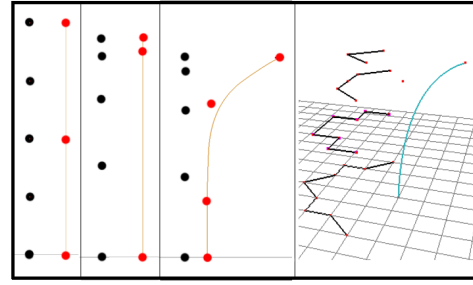**Figure 7:** *Starting 2DPP. The part inside red lines is the motif. Layer lines of the motif are highlighted.*

Figure 8: *An example of using 3D layer lines. Left: Front view, 3D layer lines are highlighted. Right: Resulting muqarnas column capital.*

3.3. Modeling

We have implemented a software tool that uses the layered structure of muqarnas while taking advantage of 2DPPs. As a starting point for creating a muqarnas design, we begin by using 2DPP as a guiding image. Since these maps can be inexact, we use them merely as a guide. On this image, modeller should specify parts of the map which are either symmetrically or exactly repeated by drawing lines on the symmetry axes (see Fig.7). These symmetry axes are used to impose exactness into the design. The axes describe a repetitive motif for the final model, where elements in this space are procedurally repeated to form the muqarnas.

Another important specification of muqarnas is the layered structure. 2DPP maps are also a useful guide for specifying the layer lines. Since the complete model is made from the repetition of motives, We only need parts of the layer line inside the motif. For making modeling easier, we use a snapping tool to help the user connect layer lines to boundaries, other layer lines and corners precisely. Although in the traditional description of muqarnas layer lines are placed on planes parallel to the ground, we provide the possibility of using 3D curves as the layer lines. With such a feature, our method provides the freedom to create new muqarnas designs beyond the traditional forms (see Fig.8).
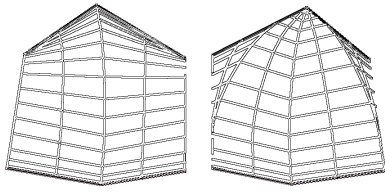
After creating layer lines, we must place each line at its desired height. The height of each layer is not specified on the 2DPP. In the case of modelling an existing muqarnas,
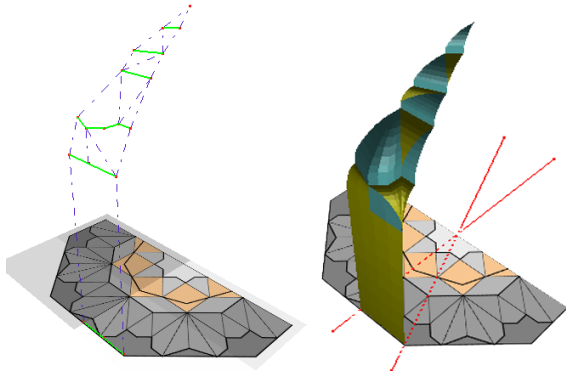

Figure 9: *Controling the height of layer lines using bspline curve. Black points are the layer lines and Red point are the control points of the curve.*

these layer lines can be placed with respect to the actual model; otherwise it can be done interactively. In the later case placing all the layers manually would be a tedious task particularly when the number of layers increase. Therefore, we provide a method to control the height of all lines using a single handle. We do this by using a B-spline curve $Q(u) = (X(u), Y(u), Z(u))$ for controlling the distribution of the layer lines. As shown in Fig.9 layers lines are mapped to points on the profile curve. The initial distribution of the corresponding points in uniform. The initial set-up for this curve is a straight line from $(0,0,0)$ to $(0,1,0)$ with a suitable number of control points (e.g. four). Each layer is mapped to a point on this curve and the height of that layer is the same as the $Y(u)$ value of the corresponding point. By reshaping the curve using its control points, we can direct the arrangement of layer lines. This enables us to place the layers in relatively proper height by approximating the curvature of the vault or dome using a B-spline curve.

To create the surface of a muqarnas, in addition to the layer lines we also need to know the correspondence between points on two adjacent layer lines. This correspondence defines the boundaries for each patch and it has an important role in the final form of the muqarnas. As Fig.10 shows, the same layer lines with different correspondence points create two different muqarnas forms. The correspondence is usually specified in 2DPP with dashed or thinner lines (see Fig.6). However, vertical correspondence lines (those that connect two layers that touch each other in 2DPP) cannot be delineated in 2DPP. We also take advantage of these connection lines in our interactive system for providing the actual correspondence. Corresponding points are connected by the user with binding lines as shown in Fig.11 (left). Since these connection lines specify the the boundaries of patches, in order to prevent twist in the resulting patches that may result from the crossing connection lines, there is a constraint on the order of points they can connect. Suppose P is the set of points on a layer line L_1 and Q is the set of points on the layer line L_2 adjacent to L_1 Assuming that connection line C_{ij} connects points P_i and Q_j and con-

Figure 10: *Different bindings results in different forms. Left and right forms have the same layer lines but different bindings.*
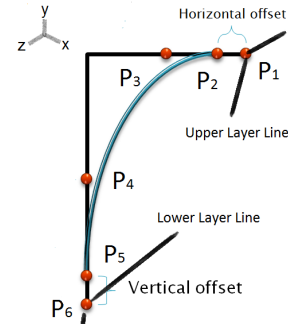

Figure 12: *Placement of control points . Blue curve shows the profile of the surface between two layer lines.*

Figure 11: *Left: binding the layers. Connection lines are shown using dash lines. Right: A completed motif*
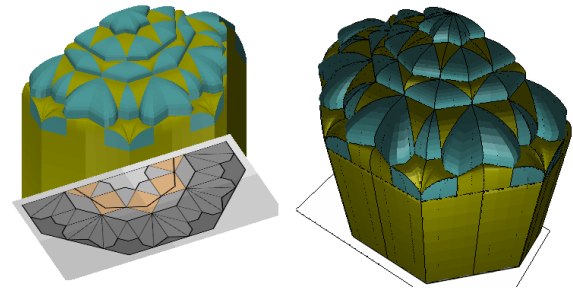

Figure 13: *Completed muqarnas vault.*

nection line C_{kl} connects points P_k and Q_l then if $i < k$ then $j \leq l$. This constraint is enforced in our software tool. Therefore, if user inputs a wrong connection order, automatic swap of the end points will maintain the integrity of the model.

Layer lines together with the connection lines form a skeleton of muqarnas. In this skeleton each layer is consisted of a sequence of cells. In order to build each layer, these cells replaced with patches that interpolate the layer lines but exhibit a controllable curvature along the connection lines. In order to find a proper geometric patch that would behave in this way, we considered the possibility of using permanence Coons patch [FH99] and cross-sectional blending [CSSJ05], however, they do not have enough flexibility to control the concavity of the patch. Furthermore, since there are sharp internal features in most types of muqarnas, it is necessary to use a surface representation that supports such features. We found that a B-spline surface which is quadratic in V direction and linear in U direction is a good choice since it interpolates U boundary curves. Moreover, it has enough flexibility for deforming the profile and supports a controllable level of smoothness.

To create these surfaces we use six control points along the height of the surface (V direction). The number of control points along the width (U direction) is equal to the number of control points on the corresponding segment of layer lines. This requires equal number of control points in the upper and lower line segments. In case they are not equal, we use knot insertion into the layer line with fewer control points.

The control points in V direction are placed as shown in Fig.12. This configuration allows us to control the curvature, vertical offset at the bottom, and horizontal offset at the top of the surface. The points P_1 and P_6 are placed on the higher and lower layer lines respectively. P_2 and P_5 control the horizontal and vertical offsets of the surface while P_3 and P_4 are used to control the curvature of the surfaces. The resulting surface has controllable curvature with horizontal and vertical offsets. Since adjacent patches share the same control points, they are connected seamlessly. Adjacent patches have the same curvature on their common edge.

The process described so far creates parts of the muqarnas between symmetry lines as shown in Fig.11 (right). These parts are considered as the motifs of muqarnas. There can be more than one motif in a muqarnas. Next, we create as many copies and mirror instances of these motifs as needed to complete the model (see Fig.13).

3.4. Editing

For interactive modeling of muqarnas, it is necessary to provide a set of convenient interactive editing tools. We have
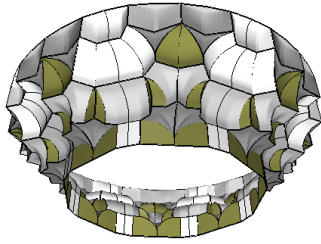
Figure 14: *A column capital designed by the implemented software*

explored several editing techniques. Some edits manipulate the whole structure of a muqarnas. for example, In case of changing the height of the structure, all the elements of muqarnas are modified. Some modifications have local effect. For instance, it is possible to change layer lines' height individually and the model is updated accordingly. Some changes are done locally but the effect is propagated to all the similar parts. For example, a user can manipulate a single muqarnas' motif. This change affects all the mirror or copy instances. We do this by defining local coordinates for every instance.

3.5. Computer generated designs

Our workflow allows us to automatically generate new muqarnas forms. Muqarnas design is the result of a symmetric configuration, its layered structure and how these layers are connected. By changing these factors and their parameters (e.g. correspondence between layer lines) we can automatically create interesting designs. Furthermore, by controlling these factors we can control the space of the muqarnas designs. Examples of these models can be found in the supplementary materials. Currently, our system can only generate radial designs with a single motif, but the idea is easily extendable to more complex designs. We start with defining an angle for the radial motif and setting the boundaries for that. Next, we sets layer lines inside the boundaries so that they do not cross each other. Then, we place the layers in ascending heights and connect them in such a way that none of the connection lines cross. These models can be used as a base design for artistic experiences with muqarnas forms.

4. Results

Using our tool, we have modeled a number of existing muqarnas buildings as well as new designs. Fig.14 shows a column capital as an example of a new design. Fig.1 shows a model of an existing building superimposed over the original image. Fig.15 shows the final textured result for the model that was used as an example through the paper. We have used images from original buildings as textures for this model. Fig.16 shows an internal view of another textured model. Each of these models has been made in less than 30 minutes.

Figure 15: *Textured model of the example used in this paper.*

5. Conclusions and Future Work

Given the geometrical complexity embodied in muqarnas designs, our proposed approach not only facilitates the reproduction of existing models but can also generate new models based on this enchanting Islamic architecture. From the guiding 2DPP to the final stage of creation, the interactive features of our system adds another dimension for enriching such structures, let alone preserving this cultural heritage. Our approach uses traditional surface definition which makes it feasible to be included in existing surface-based systems. Further control of the patches' interiors between the layers is still needed. Possibilities for future work includes exploring techniques to generate star and Persian floral patterns on muqarnas. Furthermore, we intend to study the possibility of using subdivision surfaces for generating control meshes for the underlying surfaces.

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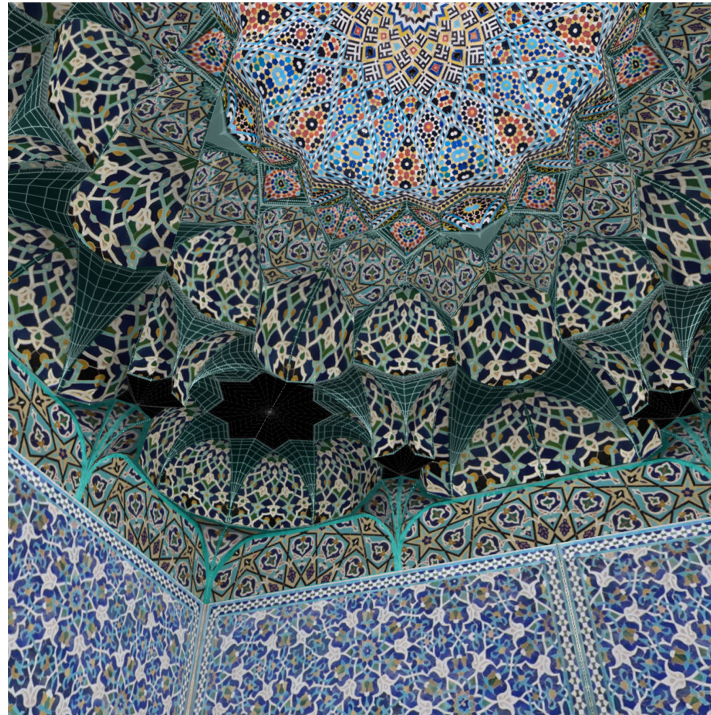
Figure 16: *Example of a model created using our software.*

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