

# 4D Surface Mesh Reconstruction from Segmented Cardiac Images using Subdivision Surfaces

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## ABSTRACT

With the advances in cardiovascular imaging technologies in recent years, 4D (3D+time) patient-specific modeling of the heart has attracted many research interests. Computational modeling approaches such as Computational Fluid Dynamics (CFD) and Finite Element (FE) have been increasingly used to quantitatively diagnose and predict cardiovascular diseases. In these methods, the geometrical reconstruction of the heart anatomy is usually an indispensable step. This work presents a robust method for reconstructing time-varying subdivision surfaces from the segmentation masks of cardiac images. We first reconstruct a 3D mesh for the first time step by iteratively fitting an initial mesh based on error and tension terms. Each subsequent time step uses the model from its previous time step as the control mesh for subdivision surface fitting. This method preserves the 1-to-1 vertex correspondence between meshes in different time steps and allows us to control the mesh quality (i.e. resolution, smoothness, and accuracy). Furthermore, in contrast to contour-based algorithms, our method can handle non-trivial topological changes such as holes and tunnels. The method has been tested on 3D and 4D datasets of different modalities (i.e. CT and MRI), resolutions, and chambers. For creating visually appealing results, we show that synthetic textures can be mapped to the 4D reconstruction due to the vertex correspondence. We also quantitatively evaluate the reconstructed 3D meshes in terms of mesh quality and conformity to the data.

## CCS CONCEPTS

• **Computing methodologies** → **Shape modeling**; • **Applied computing** → **Life and medical sciences**.

## KEYWORDS

heart surface reconstruction, subdivision surface, cardiac images

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## 1 INTRODUCTION

Cardiovascular disease is the leading cause of death worldwide. Cardiac imaging technologies provide non-invasive ways for the diagnosis, monitoring, and treatment of cardiac diseases. Various imaging technologies such as computed tomography (CT), cardiac magnetic resonance (CMR), single photon emission CT (SPECT), and echocardiography have enabled the acquisition of both 3D and 4D (time-varying) cardiac images for various purposes. With the advances in imaging techniques, the creation of personalized cardiac anatomical models has received much research attention recently [13, 22]. Patient-specific computational modeling and simulation can help us better understand cardiac functions and make clinical predictions. For instance, Computational Fluid Dynamics (CFD) and Finite Element (FE) have been increasingly used to quantitatively analyze the behavior of cardiac blood flow (hemodynamics) [16]. Moreover, personalized heart models are a prerequisite to surgical planning tools such as visual reality (VR) and 3D printing [2]. In these applications, the geometrical reconstruction of the heart anatomy, either 3D or 4D, is usually an indispensable step.

Besides being fundamental to simulation and analysis, geometric models are also important to the visualization of complex heart anatomies. In clinical practice, medical images are mostly viewed as 2D slices or cross sections. Personalized cardiac anatomic models can be used as a complementary tool in training and patient education. Furthermore, dynamic information about the heart (e.g. blood flow behavior, myocardial motion and deformation) cannot be easily obtained from 2D cross-sectional images. Constructing a time-varying heart model not only provides important insights into the anatomical information, but also helps the physicians understand and evaluate the mechanical function of the heart. Additionally, with the rapid development of automatic segmentation techniques for cardiac imagery [7], there is a growing need for both 3D and 4D patient-specific heart model reconstruction methods.

Constructing patient-specific cardiac model is challenging: the anatomy of the heart is complex and unexpected abnormalities (e.g. holes) may occur. Many approaches have been proposed over the years to reconstruct geometric models of the heart from cardiac images. Most of the studies are focused on reconstructing heart chambers and aorta as these components are usually easier to be delineated from medical images [10]. Some reconstruction methods are image-independent, which means that the geometry is extracted from prior segmentation results (e.g. contour lines, binary segmentation masks, and point clouds) [5, 17, 27–29]. Whereas others involve segmentation and image processing (e.g. registration) in the mesh construction pipeline [11, 30, 31].

More generally, surface reconstruction methods can be broadly categorized into two major types: explicit and implicit [15]. Explicit approaches (e.g. Delaunay triangulation [3] and ball-pivoting algorithm [4]) reconstruct the data as parametric or triangulated surfaces that precisely represent the object with vertex locations. Whereas implicit approaches (e.g. marching cubes [19] and radial basis function [6]) typically define the surface as implicit functions. Readers are referred to [15] for a more detailed review on 3D surface reconstruction. Regarding 4D surface reconstruction, deformable models and statistical shape models (SSM) are widely used [5, 11, 30]. One of the challenges of these methods is that they mostly rely on prior knowledge of the shape or topology for constructing the initial model. Our method differentiates from other related algorithms as it only requires a segmentation as input and works for arbitrary topology.

Subdivision plays an important role in computer graphics for producing a smooth appearance. Starting with a coarse control mesh, new mesh elements (i.e. faces, vertices, and edges) are generated based on subdivision rules, and different levels of details can be achieved. Due to the coarse-to-fine (or multiresolution) property of subdivision surfaces, they have been widely used in areas such as animation [8], Discrete Global Grid Systems [1], and sketch-based modeling [20]. The hierarchical structure of subdivision also enables multiresolution manipulation for curves and surfaces [21, 23, 24], where the change of control mesh is reflected on the refined mesh. Additionally, the ability of modeling arbitrary topologies [25] makes subdivision surfaces suitable for reconstructing complex heart anatomy.

In this paper we present a framework for extracting temporally coherent (i.e. 1-to-1 vertex correspondence) surface models from time-varying segmented cardiac images using Loop subdivision. Using a subdivision representation allows us to create a smooth approximation and to capture details at multiple scales. The method is outlined as follows (Figure 1):

- (1) From a 3D segmentation mask of one of the time frames, we construct a high-resolution mesh using marching cubes algorithm to capture all geometric details of the surface.
- (2) An initial control mesh is constructed by simplifying the high-resolution mesh, and its subdivision matrix is obtained.
- (3) A weighted least-squares system is constructed and solved for optimizing the control mesh. This step can be iteratively performed until a desirable threshold has been reached.
- (4) After the control mesh of the current frame is optimized, we use the current control mesh as an initial mesh for the next frame. Subsequent time frames can be constructed using the same approach.

The main contribution of this method is the ability to produce temporally coherent 4D cardiac meshes which balance the visual appearance and the fitting accuracy. The novel use of subdivision surfaces in the reconstruction allows us to fit the data at a controllable resolution. Subdivision is also important in tackling the problem of topological variations.

## 2 RELATED WORK

Topology is one of the main challenges of heart surface reconstruction. Depending on the specific problem, simplified heart models or

sophisticated topology handling can be used. In [5], the left heart model is divided into three parts (i.e. the LV sac, the Y-junction, and the connection between them) before the surface fitting. [28] assumes that the segmentation of a heart chamber is topologically equivalent to a sphere and ignores other structures such as valves. In [17], all delineation contours are mapped onto a single plane and construct meshes using Delaunay triangulation. A tree structure is then extracted to take into account the intra- and inter-frame topology changes. One of the limitations of this method is that it only works for parallel contours. Another challenge is to balance the data fitting accuracy and the visual appearance. In [27], the authors use thin plate splines (TSP) to iteratively fit an initial mesh to the contour lines and use butterfly subdivision and Laplacian smoothing to improve the visual appearance of the surface. However, the subdivision is followed by an independent coarsening step to regularize the vertex location. This loses the inherent coarse-to-fine hierarchy of subdivision surfaces and increases the computational time. Our method is similar to [11] which uses decimation, smoothing and remeshing to reconstruct a high-quality initial mesh and uses it to fit subsequent meshes. In comparison, our method constructs a coarser initial mesh and uses its subdivision to capture details in the data fitting. A more comprehensive review of cardiovascular modeling techniques is presented in [10].

## 3 SUBDIVISION SURFACE FITTING

In order to fit a subdivision surface to the volumetric data of a single time frame, we first construct an initial control mesh from the data, and then we model the problem as a weighted least-squares system. We fit the subdivided initial mesh to the boundary of the volumetric data by solving a linear system, from which an optimized control mesh is obtained. Figure 2 gives a high-level overview of the surface fitting process.

The first subsection introduces the datasets, followed by the details of the surface fitting process.

### 3.1 Data representation

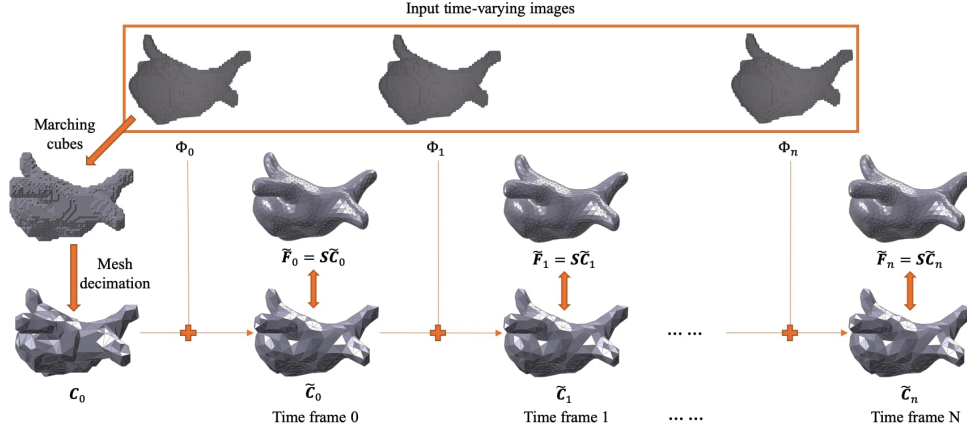
The datasets we use are segmentation masks of different heart components (e.g. LV, LA, and left heart). A segmentation mask of a 3D cardiac image can be represented by a binary function  $\Phi : \mathbb{N}^3 \rightarrow \{0, 1\}$ ,

$$\Phi(\mathbf{i}) = \begin{cases} 1 & \text{inside} \\ 0 & \text{outside} \end{cases},$$

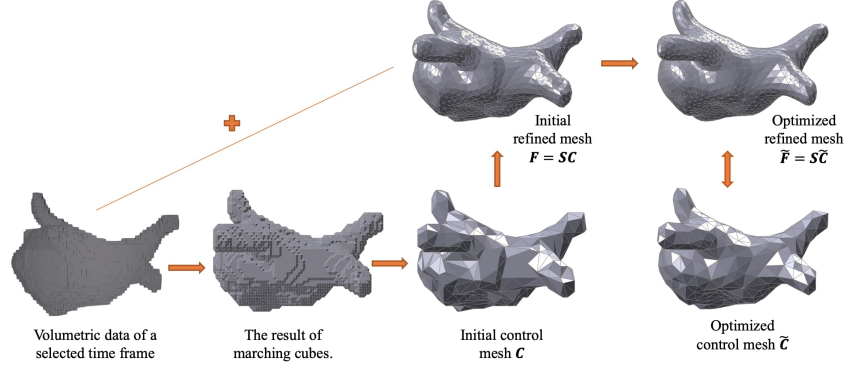
where  $\mathbf{i} \in \mathbb{N}^3$  is the index of a voxel. A signed distance transform ( $D : \mathbb{N}^3 \rightarrow \mathbb{R}$ ) is then computed from the segmentation mask to measure the Euclidean distance of a point in space to the heart surface. The distance transform  $D(\mathbf{i})$  of voxel  $\mathbf{i}$  is positive if the voxel is inside the volume and negative if outside. From the discrete distance transform, we compute  $d(\mathbf{v})$ , the distance of a vertex  $\mathbf{v}$ , using tri-linear interpolation (i.e. linear interpolation along x,y,z directions).

### 3.2 Control mesh construction

A control mesh of a subdivision surface is the coarsest level mesh that controls the geometry of the subdivision surface. To reduce the stair-stepping effect and take advantage of the hierarchical



**Figure 1: Time-varying subdivision surface construction.**  $\Phi_i$  is volumetric image of time frame  $i$ .  $C_0$  is the initial control mesh of the first time frame.  $\tilde{C}_i$  and  $\tilde{F}_i$  are the optimized control mesh and refined mesh of time frame  $i$ , respectively. The initial control mesh for time frame  $i$  is  $\tilde{C}_{i-1}$ .  $S$  is the subdivision matrix.



**Figure 2: Subdivision surface construction for the first time frame.**

structure, the control mesh should be coarse and have the same topology as the volumetric data. In order to capture topological details of the data in the control mesh, we first construct a high-resolution mesh using the marching cubes. We then apply an edge-collapsing mesh decimation algorithm to reduce the number of polygons on the dense mesh [14]. Given an appropriate decimation rate, the simplified mesh (denoted by  $C$ ) will have the same topology and will keep important geometry details of the original data (Figure 2).

### 3.3 Mesh optimization

Once the initial control mesh  $C$  is obtained, it is deformed to better fit the volumetric data. This surface fitting process is controlled by two terms: error and tension. The error term maintains the closeness of the mesh to the boundary, while the tension term maintains the fairness and smoothness of the mesh.

**3.3.1 Error term.** The error energy determines how close the current mesh is to the boundary. It acts as an external force to “pull” the vertices toward the boundary. For each vertex  $\mathbf{v}$ , we want

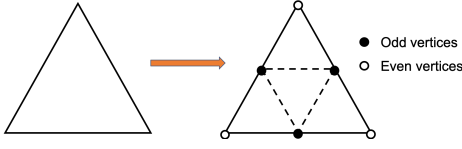
to minimize its distance to the boundary of the volume (i.e. its distance transform  $d(\mathbf{v})$ ). The per-vertex error is determined by  $e(\mathbf{v}) = \|d(\mathbf{v})\|^2$ . We define the error term as the summation of all vertex errors. The error term of a mesh  $M = (V, E)$  is  $E(M) = \sum_{\mathbf{v}_i \in V} \|d(\mathbf{v}_i)\|^2$ . Since the distance transform is a scalar field (i.e. the magnitude of the external force), we use the unit vertex normal  $\hat{\mathbf{n}}(\mathbf{v})$  to determine the direction of the external force. Assume  $V'$  is the vertex set after the mesh evolving (the edge connection is not changed during surface fitting), the error term is minimized when

$$V' = V + N(V), \quad (1)$$

where

$$N = \begin{bmatrix} d(\mathbf{v}_1)\hat{\mathbf{n}}(\mathbf{v}_1) \\ d(\mathbf{v}_2)\hat{\mathbf{n}}(\mathbf{v}_2) \\ \vdots \\ d(\mathbf{v}_m)\hat{\mathbf{n}}(\mathbf{v}_m) \end{bmatrix}.$$

**3.3.2 Tension term.** The tension energy determines how smooth and fair the current mesh is. It acts as an internal force that redistributes the vertices to ‘unfold’ the mesh and to avoid triangles that



**Figure 3: Illustration of Loop subdivision. Each triangle face on the mesh is split into 4 new faces. After face splitting, the odd vertices (i.e. the new vertices on each original edge) and the even vertices (i.e. the old vertices) will be re-positioned based on different rules.**

are too skinny and stretched. Mesh Laplacian [9] has been used for improving mesh smoothness and fairness. For each vertex, a tension force is computed as a vector from its current position to the center of its neighbors:

$$L(\mathbf{v}_i) = \frac{1}{\text{valence}(\mathbf{v}_i)} \sum_{(i,j) \in E} \mathbf{v}_j - \mathbf{v}_i.$$

The tension term of a mesh  $M = (V, E)$  is  $T(M) = \sum_{\mathbf{v}_i \in V} \|L(\mathbf{v}_i)\|^2$ . The tension term is minimized when

$$LV' = 0, \quad (2)$$

where

$$L_{ij} = \begin{cases} \frac{1}{\text{valence}(\mathbf{v}_i)} & \text{if } (i, j) \in E, \\ -1 & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

### 3.4 Surface fitting

The initial control mesh is fitted by iteratively minimizing the two constraints. The optimization is an over-determined system which is solved in a least-squares sense. In order to capture the details, we use the refined mesh to minimize the energy and to get an optimized coarse mesh.

**3.4.1 Least-squares system.** From the energy minimization equations (1) and (2), we obtain an over-determined system:

$$\begin{bmatrix} L \\ I \end{bmatrix} \mathbf{V}' = \begin{bmatrix} 0 \\ \mathbf{V} + \mathbf{N}(\mathbf{V}) \end{bmatrix}.$$

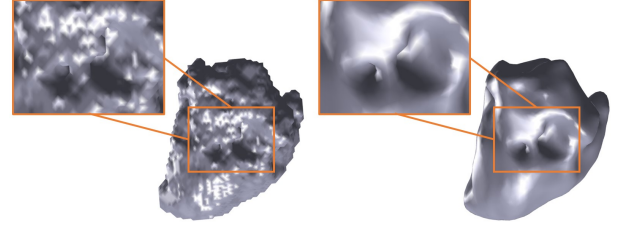
We can assign different weights  $W_T$  and  $W_E$  to the error and tension terms:

$$\begin{bmatrix} W_T L \\ W_E \end{bmatrix} \mathbf{V}' = \begin{bmatrix} 0 \\ W_E(\mathbf{V} + \mathbf{N}(\mathbf{V})) \end{bmatrix}. \quad (3)$$

The solution is the minimizer of

$$W_T^2 \|LV'\|^2 + W_E^2 \sum_i \|\mathbf{v}_i + d(\mathbf{v})\hat{\mathbf{n}}(\mathbf{v}_i) - \mathbf{v}'_i\|^2.$$

Note that  $W_T$  and  $W_E$  are constants that determines how important the tension and error terms are. Based on our experiments, the combination of  $W_T = 1$  and  $W_E = 0.25$  ensures a good visual appearance and a good fit to the data. Table 1 depicts the effect of different weights: a larger tension weight produces a smoother mesh, while a larger error weight makes the mesh closer to the data and reduces the mesh smoothness.



**Figure 4: The left atrium with a hole. Left: marching cubes mesh. Right: our method with one level of subdivision. Dataset is obtained from [26].**

**3.4.2 Loop subdivision matrix.** Loop subdivision is a subdivision scheme that is designed for triangle meshes. Loop subdivision is a generalization of cubic B-spline subdivision, and it has  $C^2$  continuity at regular vertices (i.e. valence of six). The subdivision process is composed of two steps: face splitting and vertex re-positioning. As depicted in Figure 3, each triangle face on the mesh is split into 4 new faces. The 3 new vertices on the original edges are called “odd vertices”, and the 3 old vertices are “even vertices”. Each vertex are then re-positioned based on the subdivision masks proposed in [18].

Subdivision is a coarse-to-fine approach, where the refined mesh can be represented by the product of a subdivision matrix and the coarse mesh. Let  $C$ ,  $\tilde{C}$ ,  $F$  and  $\tilde{F}$  be the vertex sets of the initial control mesh, optimized control mesh, initial refined mesh, and optimized refined mesh, respectively. Then, we have

$$\begin{cases} F = SC \\ \tilde{F} = S\tilde{C} \end{cases},$$

where  $S$  is the subdivision matrix. The subdivision matrix can be computed when the vertices are iteratively subdivided, and it can represent any level of subdivision. For instance, let  $C^0$  be the vertex set of the control mesh and  $C^i$  be the vertex set after the  $i$ -th iteration of subdivision. Then  $C^i = S_i C^{i-1}$ ,  $i = 1, 2, \dots$ , where  $S_i$  is the subdivision matrix of the  $i$ -th iteration. Thus, the vertex set after  $n$  levels of subdivision is  $C^n = S_n S_{n-1} \dots S_1 C^0 = SC^0$ , where  $S = S_n S_{n-1} \dots S_1$  represents the collective subdivision matrix of the entire process. Since the subdivision matrix is sparse, the computation can be done efficiently, and the result can be saved for subsequent time frames. From equation (3), we derive the linear system for the subdivision fitting:

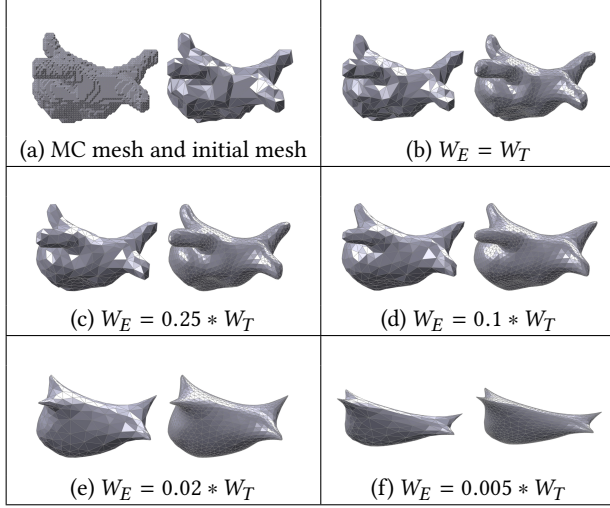
$$\begin{aligned} \begin{bmatrix} W_T L(F) \\ W_E \end{bmatrix} \tilde{F} &= \begin{bmatrix} 0 \\ W_E(F + \mathbf{N}(F)) \end{bmatrix} \\ \Rightarrow \begin{bmatrix} W_T L(F)S \\ W_E S \end{bmatrix} \tilde{C} &= \begin{bmatrix} 0 \\ W_E(F + \mathbf{N}(F)) \end{bmatrix}, \end{aligned} \quad (4)$$

where  $L$  is the mesh Laplacian obtained from the refined initial mesh, and  $\mathbf{N}(F)$  is the scaled vertex normal of the initial refined mesh. The optimized control mesh  $\tilde{C}$  is found by solving equation (4) in a least-squares sense.

## 4 TIME-VARYING SURFACE

The subdivision surface fitting method in Section 3 can be used to efficiently reconstruct time-varying datasets. As depicted in Figure

**Table 1: Demonstration for different parameters. In each of (b)-(f), the left is the control mesh and the right is the result after one level of subdivision, optimized using the corresponding weights. Dataset is obtained from [26].**



1, given a set of time-varying volumes  $\{\Phi_0, \Phi_1, \dots, \Phi_n\}$ , the goal is to construct a sequence of control meshes  $\{\tilde{C}_0, \tilde{C}_1, \dots, \tilde{C}_n\}$  such that their corresponding subdivided meshes  $\{\tilde{F}_0, \tilde{F}_1, \dots, \tilde{F}_n\}$  are optimized with respect to the volumes. Since the difference between two consecutive volumes is not significant (i.e. a slight change in the geometry and no change in the topology), we only need to construct one initial control mesh for the first time step, and then use the optimized control mesh as the initial control mesh for the optimization in the next time step. This not only helps to keep the topological constancy, but also improves the performance. The process of constructing  $\tilde{C}_i$  is as follows, where  $C_i$  denotes the initial control mesh to be fitted at time step  $i$ :

- (1) If  $i = 0$ , then construct  $C_i$  by mesh decimation, otherwise,  $C_i = \tilde{C}_{i-1}$ ,
- (2) Compute the mesh Laplacian  $L_i$  and the scaled vertex normal  $N_i$  based on the subdivided initial control mesh  $F_i = SC_i$ ,
- (3) Solve  $\begin{bmatrix} W_T L_i S \\ W_E S \end{bmatrix} \tilde{C}_i = \begin{bmatrix} 0 \\ W_E (SC_i + N_i) \end{bmatrix}$  for  $\tilde{C}_i$ .

Figure 1 depicts the pipeline for time-varying subdivision surface construction. The resulting coarse mesh sequence has vertex-to-vertex correspondence between the time steps, which can be easily interpolated to produce quality animation.

## 5 RESULTS AND DISCUSSION

Since MC is a widely used surface reconstruction algorithm in the field of medicine and since many other methods are built on top of MC [11, 30, 31], we compare our method to MC in the quantitative evaluation. We have applied our method to multiple datasets for different heart components. Table 2 compares our method and the MC method in terms of mesh resolution and least-squares error. The result is based on four datasets: normal left atrium (LA1), left atrium with a hole (LA2), left heart with the aorta (LH), and

**Table 2: Compare our method with marching cubes. Column  $\#F_{MC}$  and  $\#F_{ours}$  are the number of faces of the marching cubes mesh and our mesh, respectively. Column  $\bar{E}_{MC}$  and  $\bar{E}_{ours}$  are the average per-vertex least-squares errors.**

Data	Size	$\#F_{MC}$	$\#F_{ours}$	$\bar{E}_{MC}$	$\bar{E}_{ours}$
LA1	60*83*48	14168	2832	0.431	0.346
LA2	56*74*44	14296	5144	0.499	0.297
LH	69*32*60	8114	2592	0.981	0.906
LV	67*24*55	7944	1904	0.827	0.530

**Table 3: Evaluation of the mesh stair-stepping effect. Columns  $VP_{MC}$  and  $VP_{ours}$  are the number of total vertex pairs in the meshes constructed by marching cubes and our method. Columns  $h_{MC}$  and  $h_{ours}$  are the coplanar ratios of MC mesh and ours.**

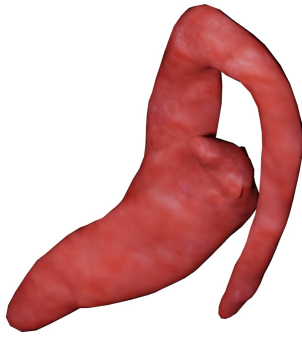
Data	$VP_{MC}$	$VP_{ours}$	$h_{MC}$	$h_{ours}$
LA1	21252	4248	45.61%	0.66%
LA2	21444	7713	41.43%	2.81%
LH	12170	3888	25.07%	0.23%
LV	11908	2856	58.58%	5.25%

one time frame from a 30-frames time-varying left ventricle (LV). Since these datasets are segmented pieces of the entire heart, we use the their bounding boxes to represent their sizes (in terms of number of voxels along x, y, z directions). From the comparison, we note that our surface has less number of faces than MC while our least-squares error remains low. The result indicates that our method is able to construct high-quality meshes (in terms of accuracy and smoothness) with lower mesh resolution. Figures in Table 1 and Figure 4 show visual comparisons between MC and our method. In Table 1, the optimized subdivision mesh ((c) right) well-approximates the volumetric data and captures important details. The optimized mesh significantly reduces the jaggedness in the MC mesh ((a) left). Figure 4 shows that the topology unchanged (i.e. the hole) in our mesh, while the surface is smoother than the MC construction.

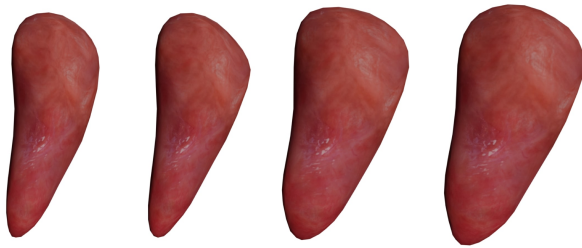
In order to quantitatively evaluate the mesh jaggedness (stair-stepping effect), we have used the quadric error metrics (QEM). The QEM method was first proposed by Garland [12] for mesh simplification. The error quadric  $Q$  for a vertex  $\mathbf{v}$  is a  $4 \times 4$  symmetric matrix that depicts the distance of a point in space to  $\mathbf{v}$ 's neighboring faces. For each valid vertex pair  $(\mathbf{v}_i, \mathbf{v}_j)$  (i.e. connected by an edge), we compute the cost of the vertex contraction  $(\mathbf{v}_i, \mathbf{v}_j) \rightarrow \bar{\mathbf{v}}$  using the quadratic error:  $E_{i,j} = \bar{\mathbf{v}}^T (Q_i + Q_j) \bar{\mathbf{v}}$ . The target point  $\bar{\mathbf{v}}$  is selected from  $\mathbf{v}_i, \mathbf{v}_j$ , and their midpoint such that it has the lowest cost. The one-ring neighborhoods of  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are considered coplanar if  $E_{i,j} < \epsilon$  (we use  $\epsilon = 1e - 06$ ). The jaggedness can be measured by the ratio of coplanar vertex pairs ( $VP_{coplanar}$ ) to the total vertex pairs ( $VP$ ):

$$h = \frac{VP_{coplanar}}{VP}.$$

A lower coplanar ratio indicates less stair-stepping effect. Table 3 compares the coplanar ratios of the marching cubes and our



**Figure 5: Reconstruction of the left heart and the aorta surface.**



**Figure 6: Four time frames of a reconstructed left ventricle animation.**

method on four datasets. The results show that the coplanar ratio of marching cubes reconstruction is higher than that of our method. Our subdivision surface significantly reduces the stair-stepping effect of the marching cubes mesh.

Our method can be used to create more visually-appealing results by attaching textures. Figure 5 shows a textured result of a reconstructed left heart surface (including left atrium, left ventricle, and the aorta). Figure 6 shows four time frames from the animation of a left ventricle during diastole. The time-varying data comes from the segmentation masks of 4D Flow MRI images over the entire cardiac cycle. The data originally consists of 30 time frames, from which we fit 30 temporally coherent meshes (i.e. having the same vertex connectivity) using the method described in Section 4. An animation with higher temporal resolution is then generated by linear interpolation. Thanks to the 1-to-1 vertex correspondence, a single synthetic texture can be mapped to all time frames.

## 6 CONCLUSION AND FUTURE WORK

We presented a method for fitting subdivision surfaces to segmented time-varying cardiac images. The method produces smooth, hierarchical subdivision surfaces while maintaining the reconstruction accuracy. The evaluation demonstrates that our surface model is able to reconstruct the segmented data with fewer polygons while reducing the jaggedness. We have also shown that it works for arbitrary topology and preserves 1-to-1 vertex correspondence in the time-varying model.

As future work, the twisting motion (or torsional motion) of the heart could be incorporated in the reconstruction. As proposed by [5], manually placed landmarks and control points can be added to take into account the torsion on the myocardium surface. Furthermore, we can adopt semi- and fully-automatic segmentation tools in the future to make it a complete pipeline for visualizing time-varying cardiac images.

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